LINEAR AND NON-LINEAR ECONOMIC MODELS OF POLLUTION

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Ph.D.

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1978

P R E F A C E

A large proportion of uncompleted theses suffer that fate not through failure to complete research begun but in writing up material prepared in draft. The present author having laboured under the difficulties of unavoidably interrupted research is only too well aware of that lamentable syndrome, and would like to take this opportunity of thanking Professor J.N. Wolfe, his supervisor, whose encouragement and dogged persistence in urging continuation of work in the face of several 'material lacunae' has been an important factor ensuring the eventual completion, and the submissions for publication to learned journals that have followed. Two sections of the thesis have in fact been favourably received by the journals, and much of Chapter 2 constitutes an article in The Manchester School (June 1976).

A B S T R A C T :

L I N E A R   A N D   N O N - L I N E A R   E C O N O M I C   M O D E L S   O F   P O L L U T I O N

1. The idea of pollution is elucidated, historically and currently; pollutants are classified analytically; the estimation of social costs and benefits from abatement are discussed.

Static micro-models of pollution involving pollution as a joint output, as a factor of production and as an input to recycling activity are developed, and comparative static analysis on each is performed. A brief consideration of the effects of exogenous technological change is made.

Pollution considered as a dynamic phenomenon using a time-dependent pollution stock and decay rate independent of the stock level. A novel social damage function with positive damage at a positive pollution threshold is postulated, and the Planning Authority's optimal control problem of maximising production benefits net of pollution costs is solved for the various possible trajectories of pollution tax and pollution stock. It is shown that society will not always prefer a decreasing quantity of pollution over time especially if it starts in the pre-threshold range of the stock. In this context it may be considered optimal to let transversality determine the final pollution stock thereby implying a zero final shadow price.

2. Various linear economic models (Leontief, Stone input-output and Activity Analysis) are expounded and developed where necessary, and their theoretical underpinnings criticised on purely economic criteria. The adaptation of these models to the study of economic-environmental interactions, specifically air pollution is examined. Several recent developments in this area are expounded and criticised and some alternative models and techniques evolved. (Activity Analysis models are developed, the concepts of commodity and industry ecology in the Stone system are analysed, and a short-run technique for shadow pricing of ecologic commodities in the absence of data on abatement is evolved.)

3. Leontief-Stone economic-environmental Input-Output is applied to a 90-sector, 12-pollutant model of the U.K. in an exercise in the methodology of pollution control. Data on pollutants initially derived from American sources is subjected to various statistical adjustment procedures based on knowledge of control efficiencies applicable to the U.K. Ecologic Impact Tables for 90 economic commodities with respect to the pollutants are calculated, and for a subset the rankings of commodities for each pollutant are tested for correlation.

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1-1 INTRODUCTION

Socrates in ancient Greek times was famed for his penetrating inquisitions into the essences of concepts. But modern philosophy concentrates on usage examples to elucidate notions, and modern science defines conceptual criteria for its own restricted and specific applications. Being denizens of the Twentieth Century we shall do as the Romans with respect to understanding the idea of pollution and thereby avoid perplexing questions about essences. There are more reasons than common consent for so doing but we shall not go into them as being irrelevant for present purposes.

Pollution is not a new phenomenon or concept (the prophet Elisha[1] cleaned up one of the first recorded instances of pollution - the sullied waters of the rivers of Jericho) though as a concept its range of application has been recently much extended. It is also certainly true to say that particular kinds of pollution have increased substantially and with much more far-reaching consequences than ever before. In the past "pollution" was a term generally applied to foul smells, smoke that got in one's eyes, on one's hair and clothing, dirtied one's windows and washing, and so on. These phenomena were, by and large, isolated events and consequences primarily of industrial production. In a predominently agrarian community, country-dwelling, pollution would be more likely to be identified with dirt in the food than algae in the streams and smoke in the air because these things occurred only to a very minor degree and preoccupations with food provision - dirty or otherwise - was the main driving force of diligence.

With the industrial revolution and the large-scale drift of population to the towns the problems of pollution became more apparent and pressing. Smoke from chimneys was, however, regarded with more than some little ambivalence by government in those days and its aspect as an index of productive activity was mainly emphasised: to be fed people required jobs; jobs created pollution in and out of work; but better alive and polluted than a corpse by starvation. Furthermore, those who organised production were always sufficiently well-off to be able to avoid its effects out of work; they lived in residential areas or outside town. Since the major influences on government prior to 1832 were the aristocracy and the business class no popular voice could be heard and people choked in silence.

The last 140 years have seen a vast expansion in population, industrial capacity and technology, the development of the welfare state and the coming of the consumer society. A consequence of this besides substantial improvements in living standards has been the pervasive increase of physical pollution and in the variety of forms it may take. An increase in the quantity of pollution, as we shall see in later sections, is a direct consequence of output expansion and increased final consumption of commodities, together with changing tastes. The changing pattern of pollution stems from technological advances generating new saleable products (which eventually end up as waste) and new processes for producing old commodities (which, because of novel input mixes generate unprecedented pollution structures). The voice of the pollution lobby can now however be heard venting counter-claims to the unmitigated pursuit of output and income growth as the sole aims of public policy and urging population control as the only solution to global eco-catastophe. The study of pollution in all its manifestations has now become, and rightly so, a significant element in our continued pursuit of the 'good life'.

1-2 CLASSIFICATION OF POLLUTION: TERMINOLOGY OF THE ANALYSIS

There are two polar cases of pollution worth mentioning for their extremity.

1. Pollutants (in the air or water or on the land) that dissipate immediately upon discharge, leaving no lasting residual beyond their instantaneous impact.

2. Pollutants that once discharged into the ambient medium remain there permanently and during which time generate measurable damage to receptors in the vicinity.

Pollutants of type (1) are called *pure flow* pollutants, of type (2) *pure stock* pollutants. The nearest empirical example of (1) would be noise pollution that inflicts damage on the ears of the receiver; of (2), certain nuclear wastes (such as Caesium 231) which have half-lives of hundreds of years and pollute the ocean almost indefinitely. Most ordinary pollutants fill the intermediate space between pure flows and pure stocks; they neither disperse immediately nor last forever, and cause measurable damage during their existence. We call them *stock pollutants*.

A further point which may be mentioned now and will be developed analytically later is the *flow input* to a stock pollutant. This consists of the emissions per unit of time of some substance(s) (physical or chemical) that gives rise to the lasting ambient concentration spearheading the stock pullutant's effect. For example, So_2 giving rise to acid rain (H_2So_4) in the atmosphere or discharges of sewage producing a BOD load in a river. Coupled with the notion of a flow input to a stock is that of the *decay rate* of a stock pollutant, a function showing how the pollution stock decays over time.

Finally, as distinct from the *mode of accumulation* (i.e. stock/ flow), there is the *impact mode* of a pollutant. This is a function showing the precise way in which the substance interacts with the receptor to cause damage. For example, noise pollution, though a pure flow pollutant, can by fracturing the eardrum cause permanent hearing loss; mercury likewise, as a stock pollutant, may remain in the environment for several years and be absorbed by man via the food chain to cause irreversible cerebral damage, paralysis and death.[3]

1-3 COSTS AND BENEFITS

1-3.1 Estimation of Costs

Much research has been expended, and fruitfully, on the biological impact of pollutant concentrations;[4] relatively little on estimating the associated economic costs of pollution.[5] This latter for the obvious reason of the difficulty in deriving adequate measures of the phenomena in question. That pollution affects individual utility functions adversely is a platitude; the question of empirical estimation hinges on the precise range and magnitude of such effects, and the usual problems accompanying the determination of demand for public goods immediately impinge. Pollution is a value-laden concept and as such is necessarily subjective in import. This of itself, of course, does not distinguish it from other subjective valuations competently handled in utility analysis; for example, preferences for classical and popular music or no music at all. The distinguishing feature lies in the quasi-public nature of its effects which are 'supplied' to large numbers of people often in the same degree. If you and I lived downwind from the bottle-kiln we both suffer roughly the same degree of soot-blight provided our geographical situation and health are similar.

What fail-safe devices have we therefore for ensuring that people really reveal their true preferences for the prospectively public supply of clean air? As Samuelson has shown,[6] each individual has an incentive under the impress of such questions to attempt to snatch some selfish benefit (a benefit not possible in the case of an appropriable, private good). An understatement of his preference for clean air is more than likely since each person obtains the same amount no matter what he pays, given that clean air is to be provided at all. Consumer marginal valuations of quasi-public good supply do seem, then, to be involved in the estimation of marginal social damages, some measure of which is necessary to determine such things as levels of tax, if any, to be applied to pollution-generating activities to obtain a social optimum of pollution.

An alternative means of pollution control to canvassing consumer opinion over marginal pollution damages, and one favoured in the U.K., is that of regulation. Here the regulating authority, directly or by mediation through central government, assumes an *a priori* knowledge of the socially optimal levels of pollution generated by a set of activities and lays legal prohibitions on those activities against discharges exceeding predetermined levels. Breaches of these requirements theoretically result in legal action by the enforcing authority and subsequent fines being imposed on miscreants. Frequently, in practice however, the authorities have been very reluctant to prosecute offenders, and fines have been set too low to act as a substantive deterrent to excessive discharge.[7] Furthermore, the costs of enforcing regulatory standards relative to a tax system are high - a point noted by Baumol and Oates [12].

1-3.2 Estimation of Benefits

The social costs of pollution are primarily a function of the pollution stock; concentrations of Ozone in a thermal inversion are directly responsible for bronchial pathology, not the tonnage of material substance actually emitted from the factory chimney or car exhaust. Social benefits, however, are often more rationally analysed in terms of tonnage discharges from which the stock accumulates. They will be expressed in terms of the benefits derived from consuming the commodities the externality-generating activity produces. Under certain conditions these benefits may be identified with the total revenue the activity gives rise to. Once social costs and benefits of production have been expressed as functions of a common quantity, say economic output, it then becomes feasible to optimise the net social benefit with respect to this common argument. At the optimum, as is well known, this yields the result that marginal social costs must equal marginal social benefits. If, in a current situation (assuming the requisite slope properties of the two functions) marginal social costs exceed marginal social benefits, then there is a welfare justification for reducing the pollution-load borne by society.[8] Such a situation betokens a resource misallocation rectifiable by transferring resources from production of commodities and services into production of clean air.

1-4 MICRO-ANALYTICS OF POLLUTION: STATICS

This section formalises and studies the analytical characteristics of various microeconomic models of pollution. We shall be considering pollution as related specifically to the production of goods and services, not to the consumption of commodities; the latter can be analysed by and large with similar conceptual apparatus. In the short-run capital is fixed and technological change absent. Our behaviour hypotheses are initially the simple and traditional profit-maximising and cost-minimising ones. Later we shall consider alternative possibilities and relax the assumptions of fixed capital and technological steady state. The necessity of distinguishing each time the two cases of regulation and taxation is obviated by noting that, in the case of a unit tax on effluent τ, and unit fine ϕ, the cost to the firm of discharges is given by

$$\tau\varepsilon \quad \text{and} \quad \phi[\varepsilon - s] \;, \qquad s \doteq \text{constant}$$

respectively, where ε = effluent discharged and s = regulatory standard. Consequently (assuming $\varepsilon - s \geqslant 0$) the marginal cost of discharges *to the firm* are simply τ and ϕ ; s has no place in the first- or second-order conditions which interest us. Thus we shall use only the term $\tau\varepsilon$ to represent externally imposed pollution costs, bearing in mind the representational ambiguity just alluded to.

The first-order conditions for a cost-minimiser expressed as ratios will be identical with those for the profit-maximiser.[9] However, in absolute terms those for the former model have only an endogenously determined *shadow* price of output (at the margin), whereas the latter will contain the exogenously determined market price of that commodity. Comparative static analysis therefore permits an exploration of the effects on inputs (and outputs) of a change in the profit-maximiser's input *and* output prices; of a change, in the case of the cost-minimiser, only of his input prices - and, derivatively, of his marginal valuation of output change.

Pollution in the context of the individual firm may be analysed in the neo-classical framework in two basic ways: discharges as a *joint product* with zero (or negative) price and a *factor of production* or costless (in the absence of taxation) input. This twofold classification has not been noted by any one writer on the subject, though individual analyses have implicitly recognised both. Baumol and Oates [12], for example, include pollution as an argument in the firm's production function the latter represented in implicit form, thus leaving the intuition open-ended; Ethridge [14] treats pollution primarily in the status of a joint product. Leontief [17], Victor [18], and Ayres and Kneese [13] by using an input-output framework permit the representation of discharges either as a primary input (un-produced, as labour), or, as *both* input and output[10] (in the role of intermediate or final commodity).

1-4.1 Discharges as a Joint Product

The profit-maximiser's objective function is:

$$\pi = pq - c(q,\varepsilon) - k \tag{1.1}$$

with p = conventional product price,

q = conventional output,

k = constant

yielding necessary conditions

$$p - c_q = 0 \tag{1.2}$$

$$- c_\varepsilon = 0 , \tag{1.3}$$

the cost function c being convex in q and ε. But the firm's costs are actually reduced by polluting more so that equation 1.3 implies the firm will maximise the benefits (in terms of cost savings) of pollution output. This formulation assumes additive separability in the costs of both outputs and that q and ε are independently variable.

The cost-minimiser will minimise

$$c = \omega\ell + \tau\varepsilon + \lambda\psi(q,\varepsilon,\ell) + \mu[q^\circ - q] + k \tag{1.4}$$

where $\psi(q,\varepsilon,\ell) = 0$ is the production function in implicit form,

ω = wage rate,

ℓ = man-hours,

λ,μ = Lagrange multipliers.

First-order conditions

$$\omega + \lambda\psi_\ell = 0 \tag{1.5}$$

$$\tau + \lambda\psi_\varepsilon = 0 \tag{1.6}$$

$$\lambda\psi_q - \mu = 0 \tag{1.7}$$

$$\psi(q,\varepsilon,\ell) = 0 \tag{1.8}$$

$$q^\circ - q = 0 \tag{1.9}$$

are obtained. Equations 1.5-7 combine to yield

$$\omega = \mu\frac{\partial q}{\partial \ell} = -\tau\frac{\partial \varepsilon}{\partial \ell} \tag{1.10}$$

$$\frac{-\tau}{\mu} = \frac{-\partial q}{\partial \varepsilon} \tag{1.11}$$

showing respectively the equivalence of the wage rate and the marginal value product of labour in producing conventional and pollution outputs, and of the ratio of output prices to the marginal rate of product substitution. ($\mu = \frac{\partial c}{\partial q}^{*} > 0$, is clearly the shadow price of a marginal increase in conventional output at equilibrium from the parametrically given level q°). $-\tau$ is the 'price' of pollution output; but since $\tau \geqslant 0$, this is an unsatisfactory aspect of the current way of looking at pollution. However, as we shall see, treating pollution as an input requires the conceptual accommodation of a negative *quantity*. Both methods have their disadvantages due to the nature of the phenomenon being studied.

Notice also that, since $\frac{\partial q}{\partial \varepsilon} > 0$, the marginal rate of product substitution has to be given a *negative* sign - thus showing, as usual, that more of one output is obtainable only by producing less of the other. The familiar production-possibility frontier of Samuelson makes sense only with a negative price of pollution.

1-4.2 Discharges as a Factor of Production

Regarding pollution now as an input to the production process profit maximisation seeks an extremum of

$$\pi = pf(\ell,\varepsilon) - \omega\ell - \tau\varepsilon - k \tag{1.12}$$

and thus

$$pf_{\ell} - \omega = 0 \tag{1.13}$$

$$pf_{\varepsilon} - \tau = 0 \tag{1.14}$$

yielding

$$\frac{\omega}{\tau} = \frac{f_{\ell}}{f_{\varepsilon}} \tag{1.15}$$

The ratio of unit factor costs is equated to the ratio of their respective marginal products (and hence the marginal rate of substitution). The assumption that the marginal product of pollution is positive can now be given intuitive economic justification. (Fuller support to the notion is also given in later discussion of recycling.[11])

Calculation of the marginal product of a factor assumes the possibility of varying that factor alone whilst holding all other inputs constant. In the general case where production is a function of both labour, capital and pollution $q = f(k,\ell,\varepsilon)$. Holding capital and labour in the firm constant we now measure the effect on output of a small change in effluent discharge. If pollution increases, then, since less labour and capital *within* the firm need to be devoted to *abatement* of pollution, resources so freed can be utilised profitably in the expansion of output. Output will thus increase. To make this clearer, suppose the firm produces steel. Now, even with fixed capital and labour, there may be possibilities of increasing output of steel if, for example, the sintering process is operated with less care for the emission of particulate matter from the furnaces. Suppose two men are required to minimise airborne particulates, though the process can still be effectively run (productionwise) with just one operator; then that extra man can be employed to help load steel onto lorries. A similar argument applies to capital: if a worker uses a broom or suction machine to sweep up metal shavings to be put back into the process, this same equipment may be used to dust or clean existing stocks of materials or floors, thus improving the cleanliness of materials used (and so their efficiency) or to brighten up surroundings in which the men work (thereby aiding labour satisfaction).

Clearly, these propositions only hold true under the assumption of transferability of resources between uses within the firm. Highly specialised labour and capital which permit no switch between alternative uses entail a marginal product of pollution that is likely to be zero. This, however, is a limiting case, and we shall assume nonzero 'switching' possibilities henceforth, committing ourselves to as few presuppositions regarding relative magnitudes as can be feasibly made.

Counter to the above train of thought it may be argued that pollution is symptomatic of thermodynamic inefficiency. Thus, although the Second Law of Thermodynamics implies that all mass-energy transformations involve waste, most pollution-generating activities are so designated because their level of waste residuals production is greater than the technological minimum. According to this argument, then, more pollution is associated with lower, not higher, production levels, thus implying a *negative* marginal product of pollution.

This argument is, however, based on a straightforward confusion. Thermodynamic efficiency is concerned with materials-energy input-output ratios in mass terms. Clearly if the average input-output ratio is increasing the marginal product of pollution is decreasing, but it manifestly does not follow from this that the marginal product of pollution is *negative*. In other words, the criticism simply consists of a dispute regarding the second-order not the first-order conditions of the problem and, moreover, has no bearing on the concavity-convexity issue.

To summarise the first-order conditions, then, profit maximisation implies the entrepreneur pollutes up to the point where the marginal value to him of effluent equals the imposed cost. In the event of no fiscal intervention ($\tau = 0$) he will extract the maximum possible revenue from this 'free good' by equating the marginal revenue product of pollution to zero.

Performing the usual comparative-static calculations the effects, at equilibrium, of marginal changes in the exogenous parameters of the above model can be examined. The relevant partial derivatives are arranged in matrix form below.

TABLE 1.1

Qualitative Comparative Statics

$$\begin{array}{c} \\ \partial p \\ \partial \omega \\ \partial \tau \end{array} \begin{array}{c} \begin{array}{cc} \partial \ell & \partial \varepsilon \end{array} \\ \begin{bmatrix} + & + \\ - & - \\ - & - \end{bmatrix} \end{array}$$

As expected, an increasing output price stimulates both labour input and pollution discharges; the two factors being complementary in the production of the increased output. This same complementarity implies that a rise in price of either factor, though obviously reducing own-input demand, also reduces demand for the other. Raising the tax rate thus not only ameliorates pollution but also causes layoffs and curtailed output. Insofar as this reduces the profit-margin below the break-even point (where fixed costs are just covered) the 'total' conditions may also be violated, thereby throwing the firm out of business and its owner into debt.

Turning now to the cost minimiser's position we have

$$\text{minimise} \quad c = \omega\ell + \tau\varepsilon + \mu[q_0 - f(\ell,\varepsilon)] + k \tag{1.16}$$

and so

$$\omega - \mu f_{\ell} = 0 \tag{1.17}$$

$$\tau - \mu f_{\varepsilon} = 0 \tag{1.18}$$

$$q_\circ - f(\ell,\varepsilon) = 0 \tag{1.19}$$

where μ is again the shadow price of output change. Except for the addition 1.19 which fixes output at q°, these necessary conditions are identical with those of 1.13-4 above where both are expressed as ratios of prices. The comparative-static matrices, however, now have a third endogenous variable (μ) and q replaces p as exogenous variable. Corresponding to Table 1.1 we have

TABLE 1.2

Qualitative Comparative Statics

$$\begin{array}{cc} & \begin{array}{ccc} \partial\ell & \partial\varepsilon & \partial\mu \end{array} \\ \begin{array}{c} \partial\omega \\ \partial\tau \\ \partial q \end{array} & \begin{bmatrix} - & + & + \\ + & - & + \\ + & + & + \end{bmatrix} \end{array}$$

Once again own-price rises depress input demands so that pollution is amenable to influence by fiscal policy. However, output being constant in this context, an increase in taxation on effluent will, so far from reducing employment actually augment it, and this for the reason that the cost-minimiser has a greater incentive than his profit-maximising counterpart to substitute one factor for another in the production of a parametrically given output level. This result therefore provides a brighter outlook on the employment implications of effluent control - provided we once again bear in mind the necessity for the total conditions being met. In times of squeezed profit-margins this latter clause may not of course be validated, but such conditions will equally affect profit maximising enterprises so the differential 'social' advantage remains.

The remaining elements in this matrix (viz. of row and column three) demonstrate a positive association between, respectively, marginal changes in the exogenously given output level and input-demands (pollution increases, as expected, with an increased output), and marginal changes in input prices on the shadow price of output (the association being positive due to the fact that the cost of a fixed output increases with increased input prices; a pollution-tax increase therefore raises the shadow price of output, - identified with the marginal cost of deviation from its parametrically specified value).

1-4.3 Discharges and Materials Balance

Consider a manufacturer whose short-run subjectively perceived production function is given by $q = f(\ell)$. His costs consist simply of wages-plus-materials and are proportional to the quantity of labour employed. Unit materials cost will be constant if workers use a constant amount of material per man-hour worked and materials can be bought at fixed prices. Thus, as a profit-maximiser he will maximise

$$\pi = pf(\ell) - [\omega + m]\ell \qquad (1.20)$$

where

ω = wage (parametrically given)

m = unit materials cost

The necessary condition for a maximum is obviously

$$pf_{\ell} - [\omega + m] = 0 \qquad (1.21)$$

Economic input and output are measured in this context in quite disparate units: labour in man-hours, product in (say) volume terms. Suppose now, that, due to the process operated producing effluent and inflicting social damage, the authorities impose a unit-tax on the firm's discharges. The entrepreneur has an incentive to measure the various quantities of pollution produced by employing different quantities of labour for each level of output. One way of performing such a calculation is to employ the Materials Balance Principle (MBP), an application of the law of conservation of matter.[12] Symbolically, if

α = mass per unit of output,

μ = mass of materials used per man-hour,

M = mass of capital equipment,

\- all these quantities being constants, then the MBP asserts the equality

$$\alpha f(\ell) + \varepsilon - M - \mu\ell = 0 \qquad (1.22)$$

must *identically* hold for all vectors (ε, ℓ) . The maximand now becomes

$$\pi = pf(\ell) - [\omega + m]\ell - \tau\varepsilon - \lambda[\alpha f(\ell) + \varepsilon - M - \mu\ell] \tag{1.23}$$

optimisation of which entails

$$pf_\ell - [\omega + m] - \lambda[\alpha f_\ell - \mu] = 0 \tag{1.24}$$

$$- \tau - \lambda = 0 \tag{1.25}$$

$$- [\alpha f(\ell) + \varepsilon - M - u\ell] = 0 \tag{1.26}$$

Clearly $-\lambda \geqslant 0$ as $\tau \geqslant 0$. Thus the shadow price of 'ecologic efficiency' $\lambda = \frac{\partial\pi}{\partial S}$, where $S \equiv \varepsilon - M - \mu\ell$ or $S \equiv \alpha f(\ell) - M - \mu\ell$ becomes positive only if an external cost is imposed on the firm to limit its effluenct. Only in this situation is the ratio of total effluent to input-matter, or the ratio of total economic output to input-matter, of any significance. It is easily verified that the second order conditions for 1.23 to be a maximum are satisfied.

1-4.4 Recycling versus Discharge of Production Residuals

So far we have examined the relation between pollution and production considering abatement only in its role as negative pollution; a question legitimately raised is, Where does the material abated actually end up? Matter cannot be destroyed and therefore the question has a poignant relevance to specification. In our equation for the MBP (1.22) we assumed that total residuals from production were discharged into the environment; the firm's only option for reducing pollution was to decrease mass inputs.

But this (in the short-run framework) meant reducing his input of labour, and so reducing economic output. An alternative and more general formulation of the firm's MBP will allow the firm to accumulate mass stocks of residuals from the production process and/or allow the tranformation of these into saleable output. This latter process we shall call *recycling*.

Recycling in the static theory of the firm has been treated in a paper by Ethridge [14]. His model merits discussion and using the current notation the ideas can be expressed as follows.

Let γ be the total quantity of 'waste' residuals generated by a productive process. Then, if μ is the quantity of bi-product produced by the process, and ε waste discharges, we have the identity

$$\gamma \equiv \mu + \varepsilon \tag{1.27}$$

Production of the principle product of the firm of the bi-product and of the total residuals are functions of the same quantities of capital and labour:

$$q = F(k,\ell) \ , \quad \mu = \psi(k,\ell) \ , \quad \gamma = uq = uF(k,\ell) \tag{1.28}$$

where q is principal product, u = constant .
Thus, total residuals are proportional to economic output.[14]
From equations 1.27 and 1.28 we obtain an expression for discharges of waste:

$$\varepsilon = uF(k,\ell) - \psi(k,\ell) \tag{1.29}$$

The entrepreneur maximises an objective function

$$\begin{aligned}\pi &= p_q q + p_\mu \mu - \rho k - \omega\ell - \tau\varepsilon \\ &= p_q F(k,\ell) + p_\mu \psi(k,\ell) - \rho k - \omega\ell - \tau[uF(k,\ell) - \psi(k,\ell)]\end{aligned} \tag{1.30}$$

where p_q , p_μ are the (parametric) prices of principal and bi-products. Necessary conditions are

$$p_q F_\ell + p_\mu \psi_\ell - \omega - \tau[uF_\ell - \psi_\ell] = 0 \tag{1.31}$$

$$p_q F_k + p_\mu \psi_k - \rho - \tau[uF_k - \psi_k] = 0 \tag{1.32}$$

But since there is no guarantee that 1.30 is a concave function there is no certainty that the sufficiency conditions hold; the Hessian, as can be seen from a few simple calculations, may well turn out not to be negative definite. In fact, Ethridge seems to have been misled by the specious generality of his own notation, (viz., the representation of k,ℓ (etc.) by the general term $x_i, i = 1, \ldots n$) into treating the inputs x_i as if they were one. Of course, in the short-run (as in models of the previous section) such an assumption can be legitimately made; however, this is plainly not Ethridge's intention. If we in fact assume $n = 1$, e.g. $x_1 = \ell$ (as before) then the necessary conditions reduce to one, viz.

$$p_q F'(\ell) + p_\mu \psi'(\ell) - \omega - \tau[uF'(\ell) - \psi'(\ell)] = 0 \tag{1.33}$$

Yielding the second-order condition

$$p_q F'' + p_\mu \psi'' - \tau[uF'' - \psi''] < 0 \tag{1.34}$$

which holds good if

$$p_q - \tau u \geqslant 0 \quad \text{or} \quad \left|[p_q - \tau u]F''\right| < \left|[p_\mu + \tau]\,\psi''\right| \tag{1.35}$$

Using the implicit function relationship here we get

$$\frac{\partial \ell}{\partial p_\mu} = \frac{-\psi'}{[p_q - \tau u]F'' + [p_\mu + \tau]\,\psi''} \tag{1.36}$$

But from 1.34-5 the denominator is < 0 , so

$$\frac{\partial \ell}{\partial p_\mu} > 0 \tag{1.37}$$

at equilibrium. Since labour here enters jointly into principal production and into bi-production an increase in the price of the bi-product stimulates both economic *and* recycling activity at once. Furthermore

$$\frac{\partial \ell}{\partial \tau} = \frac{uF' - \psi'}{[p_q - \tau u]F'' + [p_\mu + \tau]\,\psi''} \tag{1.38}$$

$$\gtrless 0 \quad \text{as} \quad uF' \lesseqgtr \psi' \tag{1.39}$$

at equilibrium. Thus, as Ethridge states (and on the assumption that 1.35 holds), an increase in the pollution tax will stimulate both economic and recycled output: positively, if the marginal pollution product (viz. uF') is *less* than the marginal product of labour in recycling; and negatively if greater. In so doing the firm's resource-mix would alter in the situation where more than one factor is distinguished.

Ethridge's theory of recycling, though possessing certain virtues of simplicity has countervailing defects. The main deficiency lies in the assumption of a totally integrated production process. Recycling arises from production in such a way that the former output, though called a "bi-product", and intuitively therefore of a subsidiary nature, must simultaneously rise and fall in level with the main or principal commodity. Thus, if bi-production becomes unprofitable and stops $(\ell = k = 0)$ this means principal production immediately grinds to a halt $(q = \mu = 0)$. Inputs of labour and capital in this framework give rise simultaneously to both production and recycling; it being impossible to separate into additive quantities (say) hours of labour spent in producing refined oil and sulphur bi-product because each man always does two jobs at once.[15] In practice, however, the dog and its tail rarely assume identical status: recycling as a secondary product to the main activity of the firm may become unprofitable as a result of falling output prices[16] but this simply results in a reduced output of recycled commodity and a corresponding increase in pollution since inputs into the two processes are not 'simultaneous' in Ethridge's sense. In other words, the two production processes are separable.

Secondly, Ethridge's integrated-process approach does not enable us to analyse the pollution structures of the production and recycling processes separately. It is important to emphasise (see Russell and Spofford [16]) that recycling itself is not an environmentally costless activity; in its capacity as a mass-energy transformer it inevitably generates material residuals requiring disposal. Theoretically, of course, there is no reason why there should not be an almost infinite series of interlocking processes, each recycling material residuals of an earlier process in the chain. Such a sequence might be 'limited' by convergence (if each subsequent activity produced less pollution than the previous one thereby supplying less and less material input) and/or by relative prices.[17]

From the foregoing criticisms it seems desirable to develop a model that will circumvent the hypothesis of an entirely integrated production process and permit analysis of the pollution effects of both production and recycling activity. To do this it is necessary, in order to avoid bewildering complexities, to sever the recycling chain after its second link, and to assume both production and recycling produce the same kind of residual. Thus we consider a firm with one production and one recycling department, producing outputs q_1, and q_2, and discharges ε_1 and ε_2. Each have their own separate inputs of labour and capital, ℓ_i and k_i, purchased at identical, exogenously given prices, ω and p. The production department generates a total residuals load γ_1 which may be either discharged/or recycled:

$$\gamma_1 \equiv \varepsilon_1 + \mu_2 , \tag{1.40}$$

μ_2 being the quantity of materials inducted into the recycling department. The firm's production and recycling functions F^1 and F^2, assumed concave and separable (i.e. all cross-partials zero, except $F^1_{\varepsilon_1\gamma_1} > 0$.) are written

$$q_1 = F^1(k_1, \ell_1, \gamma_1, \varepsilon_1) \quad (1.41)$$

$$q_2 = F^2(k_2, \ell_2, \mu_2, \varepsilon_2) \quad (1.42)$$

The profit-maximising firm subject to parametric output prices will maximise an objective function

$$\pi = \Sigma p_i q_i - \omega \Sigma \ell_i - \rho \Sigma k_i - \tau \Sigma \varepsilon_i - B \quad (1.43)$$

with B = constant. The necessary conditions are

$$p_i F^i_{k_i} - \rho = 0 \qquad i = 1,2 \quad (1.44)$$

$$p_i F^i_{\ell i} - \omega = \bar{0} \qquad i = 1,2 \quad (1.45)$$

$$p_1 F^1_{\ell_1} + p_2 F^2_{\mu_2} = 0 \quad (1.46)$$

$$p_1 [F^1_{\gamma_1} + F^1_{\varepsilon_1}] - \tau = 0 \quad (1.47)$$

$$p_2 F^2_{\varepsilon_2} - \tau = 0 \quad (1.48)$$

Considering the firm-optimal quantities of pollution $(\varepsilon_1, \varepsilon_2)$, we have

$$p_1 [F^1_{\gamma_1} + F^1_{\varepsilon_1}] = p_2 F^2_{\varepsilon_2} = \tau \ (\geqslant 0) \quad (1.49)$$

The firm will allocate resources so that the marginal revenue products (MRP's) of discharges are equated in both departments and equivalent to the tax rate.[18]

Comparing next the relationship of pollution and recycling at the margin we find that

$$p_1 F^1_{\varepsilon_1} = p_2 [F^2_{\varepsilon_2} + F^2_{\mu_2}] \cdot \qquad (1.50)$$

The firm will allocate its residuals between production and recycling (including materials of recycling discharge) so as to equate the marginal revenue products in the two departments. Notice that in this context the MRP of production discharges is equated to the *sum* of the MRP's of recycling discharges and recycled materials. This feature of equilibrium makes salient the principle that residuals from production and from recycling now assume the status of valuable resources: because of the identity 1.40, the more material discharged the less available for transforming into saleable commodities; and, derivatively, the less available for discharging in the process of recycling. Thus, in considering the effect of a marginal change in effluent the entrepreneur will bear in mind the implications for revenue not merely from abatement avoidance, but also from foregoing recycling possibilities.

The same equilibrium condition also adds support to the hypothesis of positive marginal products of pollution. For it is impossible to increase (*cet.par.*) the output of recycled product without increasing the quantity of materials recycled $(F^2_{\mu_2} \geqslant 0)$. Negative marginal products of discharge would imply that the pollution content of a unit of recycled product was greater than the residuals recycled. If this proposition were valid (as may occasionally arise[19]) recycling processes would be characterised by 'dirty technology'. The present analysis assumes the contrary.

Finally, conditions 1.44, 1.45 require no more additional comment than the economically obvious: the MRP's of capital and labour in both departments equate to respective factor prices.

Comparative statics, under the separability assumption of the model prove to be quite simple. The profit function being concave automatically satisfies the condition of negative definiteness we require of it. The Hessian of the system is block diagonal thus easily facilitating a reduced form. Below is outlined the qualitative Jacobian relating marginal changes in exogenous to endogenous variables:

TABLE 1.3

Qualitative Comparative Statics

	$\partial \ell_1$	$\partial \ell_2$	∂k_1	∂k_2	$\partial \mu_2$	$\partial \varepsilon_1$	$\partial \varepsilon_2$
∂p_1	+	0	+	0	?	+	0
∂p_2	0	+	0	+	+	-	+
$\partial \omega$	-	-	0	0	0	0	0
∂p	0	0	-	-	0	0	0
$\partial \tau$	0	0	0	0	+	-	-

The signs on conventional inputs (capital and labour) are as expected with a rise in product price engendering additional factor demands to satisfy a newly profitable output expansion, both in the principal and recycled product. Likewise, an increase in factor costs via changing wage and capital goods' prices depresses output and, indirectly, input demands. The signs of the partials relating economic and ecologic variables are, however, of greater interest to us. Discharges from production (ε_1) are stimulated by an increase in the principal product price and depressed both by an increase in the price of recycled output and an increase in the unit pollution tax. Whilst, as we have seen, recycling activity is fostered by an increasing price of recycled output, from purely qualitative considerations it is not possible to deduce the impact of a marginal increase in the conventional product price on this output; the sign of the effect depends on the relative magnitudes of marginal products of recycled and discharged (production) residuals, in proportion to their rates of change. (For example, if the marginal product - and so the marginal revenue - from recycling is dominant, $\frac{\partial \mu_2}{\partial p_1} > 0$; if the marginal product - and so the marginal revenue - of discharges preponderates, then $\frac{\partial \mu_2}{\partial p_1} < 0$) . We may also conclude that recycling is fostered by a rising tax on pollution, which, by raising the input cost of this factor (and thus choking off demand) simultaneously enhances its relative profitability. Although any stimulation of recycling output in response to price increases (p_2) must exacerbate pollution from recycling (ε_2) , this will rarely lead to an overall increase in pollution ($\varepsilon_1 + \varepsilon_2$) since there is an associated nagative impact on production discharges (ε_1) .

Total discharges are in any event amenable to fiscal influence implying that the pollution load of the firm may always be regulated to socially optimal levels.

Reviewing the assumptions underpinning our analysis *viz à viz* Ethridge's, it should be obvious from equations 1.41-2 that a corner solution in which $k_2 = \ell_2 = \mu_2 = \varepsilon_2 = 0$, implying $q_2 = 0$, does not imply $q_1 = 0$, which is plainly a more logical outcome in the majority of cases to be met with in reality. Such a solution does not mean that in equilibrium the entrepreneur's only recourse in pollution control is to reduce output; clearly he can start up his recycling plant if price conditions once again become propitious. For the moment, however, with prices low or costs high his recycling department must stand idle.

1-4.5 Technological Change

Suppose a production function in which discharges are simply proportional to capital input:

$$\varepsilon = sk , \qquad s = \text{constant } (> 0) ; \qquad (1.51)$$

then if the production relationship is written $q = f(k,\ell)$ profits are

$$\pi = pf(k,\ell) - \rho k - \omega\ell - \tau\varepsilon \qquad (1.52)$$

yielding first order conditions

$$\rho f_k - [\rho + s\tau] = 0 \quad (1.53)$$

$$pf_\ell - \omega = 0 \quad (1.54)$$

Here pollution is treated as a factor of production and in equation 1.53 the environmental cost of capital input within the firm is allocated specifically to that economic input. Less capital will in consequence be emplbyed if $\tau > 0$ than if $\tau = 0$. Technological change may supervene by altering the whole form of the production function and the pollution-capital interdependence (say, making it nonlinear) or by causing simply an upward or downward shift in the pollution coefficient s. An upward/downward variation in s in the context of an environmentally orientated fiscal policy $(\tau > 0)$ has the same effect as an increase/decrease in the pollution tax (see Table 1 above) since it operates to increase/decrease s. The upshot is automatically to ameliorate pollution via reduced capital and labour input. But if $\tau = 0$ it is obvious that changes in s can bring about no financial impact on the firm, and therefore are environmentally 'unaccountable' - whether they be for social good or ill.

1-5 MICRO-ANALYTICS OF POLLUTION: DYNAMICS

In section 1-2 the idea of a flow input to a stock pollutant was broached. So far the discussion of pollution control has implicitly centred on tax/regulation policy related to such flow inputs. It may be possible to calculate a flow input tax level sufficient to achieve a socially optimal level of the resultant pollution stock (see e.g. Baumol and Oates [12] for a discussion of iterative techniques for accomplishing this desideratum). However, it is also possible to determine, within the context of a planning authority's decision-making framework, a shadow price of the pollution stock itself. In the nature of things the pollution stock is a time-dependant phenomenon. This implies, where time variations are significantly large, a non-constant time path for the related shadow price.

Several analyses of the criteria for determining an optimal division of production residuals between recycling and discharge where the pollutant in question has a cumulative impact on the environment have appeared recently. Models used, have however, been developed either in a macro-economic setting using a Ramsey or other large-sector type growth model (see, e.g. Keeler, Spence and Zeckhauser [20], Plourde [21], Smith [22] and D'Arge and Kogiku [23]) or have retained a static firm-theory framework. (See Ethridge [14] and Rose-Ackerman [19].) A paper by Førsund [24] provides an intermediate between these extremes by employing a micro-economic approach in a dynamic context. However, Førsund bases his analysis entirely on cost functions (thus precluding the possibility of investigating production-function relationships amongst economic and ecologic variables) and has no explicit discussion of recycling.

These theoretical lacunae leave room for a dynamic theory of the interrelationships between recycling and stock pollution in the planning context.

1-5.1 Equations for the Pollution Stock

Consider first the relationship between the stock of pollution and flow inputs from production. Let the pre-existing stock of pollution be denoted by $\overline{S}$; if emissions in period 0 are $\varepsilon(0)$, and the rate at which pollution decays from one period to another is given by α (assumed independent of the pollution stock) we have an expression for the stock of pollution at the beginning of period 0 :

$$S(0) = [\overline{S} + \varepsilon(0)] \tag{1.55}$$

and at the beginning of period 1:

$$\begin{aligned} S(1) &= [\overline{S} + \varepsilon(0)](1 - \alpha) + \varepsilon(1) \\ &= S(0)(1 - \alpha) + \varepsilon(1) \end{aligned} \tag{1.56}$$

Generalising to the t^{th} period we get

$$\begin{aligned} S(t) &= \varepsilon(t) + \varepsilon(t-1)(1 - \alpha) + \ldots \varepsilon(0)(1 - \alpha)^t \\ &= \sum_{i=0}^{t} \varepsilon(t-i)(1 - \alpha)^i \\ &= \sum_{i=0}^{t} \varepsilon(i)(1 - \alpha)^{t-i} \end{aligned} \tag{1.57}$$

where $\varepsilon(0)$ is now defined as the flow of period 0 *plus* the inherited stock of previous periods. In continuous terms this formula becomes:

$$S(t) = \int_{o}^{t} \varepsilon(i) e^{-\alpha(t-i)} di \tag{1.58}$$

Differentiating with respect to time yields

$$\dot{S}(t) = \varepsilon(t) - \alpha S(t) \tag{1.59}$$

Clearly the condition for a time-invariant pollution stock is

$$\varepsilon(t) = \alpha S(t) \tag{1.60}$$

which, roughly, says that the stock of pollution will only increase if emissions in any period exceed the environment's assimilative capacity, i.e. its ability to dissipate and render harmless the accumulated toxic wastes, of that period.[20] If emissions at the beginning of any period exceed the quantity of the pollution stock from the previous period that has decayed away the pollution stock is increasing over time. Likewise, it is decreasing if emissions in any period are less than the quantity of the previous period's stock that has decayed. In general, of course, we will find the pollution stock increasing, constant and decreasing in various time-subintervals over the period considered as a result of changing outputs and inputs to firms in response to variations in relative price and cost conditions - and, in the longer run, in response to changing technology.

It is important to notice that equation 1.60 can be satisfied by an infinity of values of ε and S ; society is, however, by no means indifferent between high and low S ; thus the equation provides no criterion of optimality in itself.

To determine the optimal value of S we must formulate a functional relationship between the costs and benefits it or its flow input give rise to.

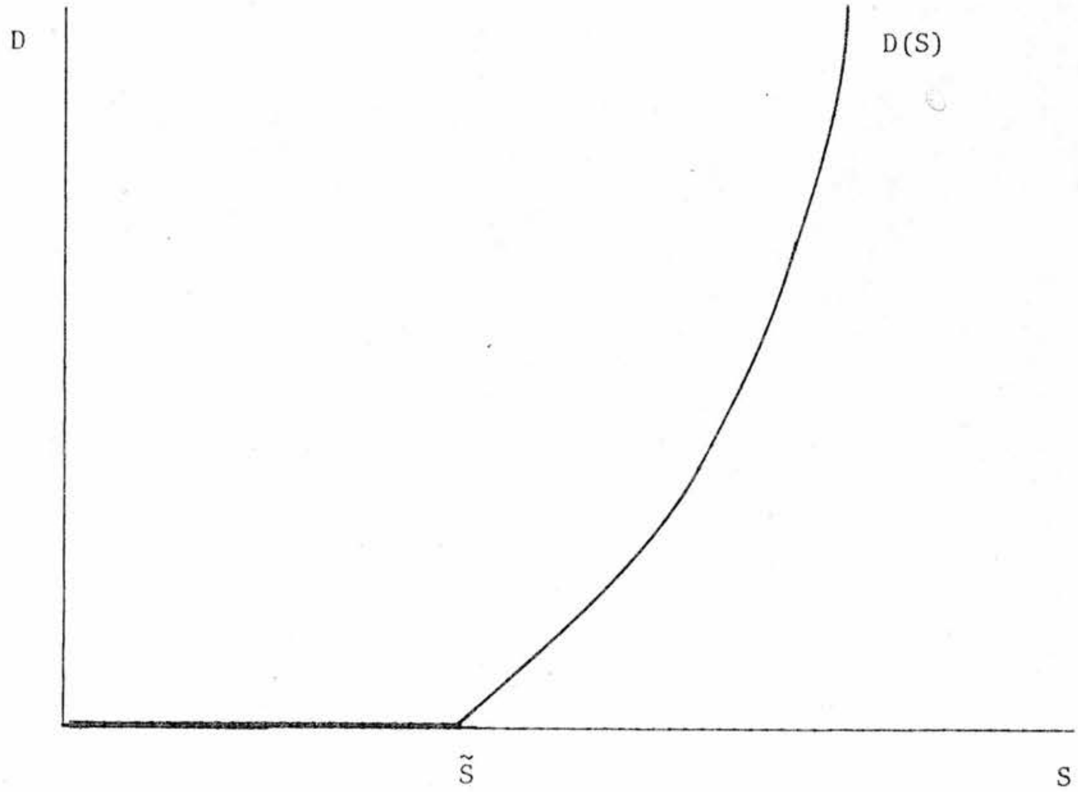
1-5.2 Social Damage Function

Consider, then, a social damage function, D(S), having the following analytical characteristics:

$$\left.\begin{array}{rl} D = 0\ ; & S \leqslant \tilde{S}\ , \quad \tilde{S} > 0 \\ D' = 0\ ; & s \leqslant \tilde{S} \\ D\ ,\ D'\ ,\ D'' > 0\ ; & s > \tilde{S} \\ D''' = 0 & \end{array}\right\} \qquad (1.61)$$

The function is represented graphically below:

Figure 1



Thus, below some threshold level, $\tilde{S}$, the stock of pollution causes no perceptible damage to receptors in the vicinity.[21] After this level is exceeded however social damage becomes positive, increasing with an augmented marginal rate.

1-5.3 Planning Authority's Control Problem

We now turn to formulate the Planning Authority's control problem.

Assume that each firm in the region under consideration possesses a production department and *potentially* a recycling department (it may be always operated at zero level). Each firm has production function relationships given by our model 1.41-2 above except that now instead of a flow of capital we can think of it as being progressively accumulated by successive purchases. Our capital accumulation formulae can therefore be written as

$$\dot{K}_1 = c_1 - \delta K_1 \qquad (1.62)$$

$$\dot{K}_2 = c_2 - \delta K_2 \qquad (1.63)$$

where K_1 , K_2 are the typical firm's capital stocks in production and recycling respectively; c_1 , c_2 are capital purchases; and δ the (physical) rate of depreciation of capital equipment.

All variables in the typical firm will now be functions of time, — though (for convenience) we shall omit the time-arguments, assuming them understood.

If there are n firms in the planning region the Authority can be considered as maximising the function

$$J = \int_0^T \left\{ \sum_{j=1}^{n} \pi_j - D(S) \right\} e^{-\gamma t} dt \qquad (1.64)$$

which is the discounted integral of net benefits from production over the time period $[0, T]$, where

$$\pi_j = \sum_{i=1}^{2} p_i q_{ij} - \rho \sum_{i=1}^{2} c_{ij} - w \sum_{i=1}^{2} \ell_{ij} \qquad (1.65)$$

represents the jth firm's profit function. 1.64 is maximised subject to equations 1.59, 1.62 and 1.63. Initially we shall assume that there are specified pollution and capital stocks at the terminal date T, $S(T)$, $K_{1_j}(T)$, and $K_{2_j}(T)$.

Thus formulated our problem is a fixed end-point optimal control problem and can be solved by invoking the Pontriagin Maximum Principle (see Hadley and Kemp [25], p.291). Form the current-value Hamiltonian

$$H = \left[\sum_{j=1}^{n} \pi_j - D(S) \right] e^{-\gamma t} + \sum_{j=1}^{n} \lambda_{1_j} \left[c_{1_j} - \delta K_{1_j} \right] e^{-\gamma t}$$
$$+ \sum_{j=1}^{n} \lambda_{2_j} \left[c_{2_j} - \delta K_{2_j} \right] e^{-\gamma t} + \eta \left[E - \alpha S \right] e^{-\gamma t} \qquad (1.66)$$

where λ_{1_j}, λ_{2_j} and η are Lagrange Multipliers and

$$E = \sum_{j=1}^{n} (\varepsilon_{1_j} + \varepsilon_{2_j}) .$$

Necessary conditions for a maximum are:

$$\dot{\lambda}_{1_j} = -\frac{\partial H}{\partial K_{1_j}} + \gamma\lambda_{1_j}$$

$$= -p_1\frac{\partial F^{1j}}{\partial K_{1_j}} + \lambda_{1_j}\left[\delta + \gamma\right] \tag{1.67}$$

$$\dot{\lambda}_{2_j} = -\frac{\partial H}{\partial K_{2_j}} + \gamma\lambda_{2_j}$$

$$= -p_2\frac{\partial F^{2j}}{\partial K_{2_j}} + \lambda_{2_j}\left[\delta + \gamma\right] \tag{1.68}$$

$$\dot{K}_{1_j} = c_{1_j} - \delta K_{1_j} \tag{1.69}$$

$$\dot{K}_{2_j} = c_{2_j} - \delta K_{2_j} \tag{1.70}$$

$$\frac{\partial H}{\partial c_{1_j}} = -\rho + \lambda_{1_j} = 0 \tag{1.71}$$

$$\frac{\partial H}{\partial c_{2_j}} = -\rho + \lambda_{2_j} = 0 \tag{1.72}$$

$$\frac{\partial H}{\partial \ell_{1_j}} = p_1\frac{\partial F^{1j}}{\partial \ell_{1_j}} - w = 0 \tag{1.73}$$

$$\frac{\partial H}{\partial \ell_{2_j}} = p_2\frac{\partial F^{2j}}{\partial \ell_{2_j}} - w = 0 \tag{1.74}$$

$$\dot{\eta} = -\frac{\partial H}{\partial S} + \gamma\eta$$

$$= D'(S) + \eta\left[\alpha + \gamma\right] \tag{1.75}$$

$$\dot{S} = E - \alpha S \tag{1.76}$$

$$\frac{\partial H}{\partial \varepsilon_{1_j}} = p_1\left[\frac{\partial F^{1j}}{\partial \gamma_{1_j}} + \frac{\partial F^{1j}}{\partial \varepsilon_{1_j}}\right] + \eta = 0 \tag{1.77}$$

$$\frac{\partial H}{\partial \varepsilon_{2_j}} = p_2\frac{\partial F^{2j}}{\partial \varepsilon_{2_j}} + \eta = 0 \tag{1.78}$$

$$\frac{\partial H}{\partial \mu_{2_j}} = p_1\frac{\partial F^{1j}}{\partial \gamma_{1_j}} + p_2\frac{\partial F^{2j}}{\partial \mu_{2_j}} = 0 \tag{1.79}$$

$$K_{ij}(0) = K_{ijo} \tag{1.80}$$

$$S(0) = S_o \tag{1.81}$$

Equations 1.77-79 imply

$$\eta = p_2\frac{\partial F^{2j}}{\partial \mu_{2_j}} - p_1\frac{\partial F^{1j}}{\partial \varepsilon_{1_j}} = - p_2\frac{\partial F^{2j}}{\partial \varepsilon_{2_j}} \lesseqgtr 0 \tag{1.82}$$

Since we want η to function as a shadow price, define

$$p_3 = -\eta (\gtrless 0) \tag{1.83}$$

1-5.4 The Optimal Trajectory

Equations 1.67-1.77 must hold at every instant along the optimal trajectory. Consider a subset of these conditions, namely equations 1.71-1.74 and 1.77-1.78. We can now establish the effects on the variables λ_{ij} , c_{ij} , K_{ij} , ℓ_{ij} , μ_{2_j} , ε_{ij} of small changes in the shadow price p_3 and of the other variables exogenous to the firm p_1 , p_2 , ρ , w .[22] This is done by a simple comparative static analysis identical with that performed for the static one-firm model of equation 1.43 above. The implications are as expected, being identical with those of Table 1.1 if τ is replaced by $-p_3$ and ρ by $\rho[\delta + \gamma]$. Since as mentioned the necessary conditions must hold at each instant along the optimal trajectory interpretation of the procedure is that we are considering small deviations from the optimal path *at any moment in time*.

We turn now to the analysis of the optimal time path of stock pollution by means of Phase Diagrams. The reader will notice that the results obtained are similar to those of Førsund [24] but certain differences arise due to the forms of damage and benefit functions employed and also the interpretation and implications of these conclusions are given additional import by our explicit representation of recycling.

First of all, we can deal with a slightly spurious dynamic element in the capital stock accumulation equations. From the necessary conditions

$$- p_1 \frac{\partial F^{1j}}{\partial K_{1_j}} + \rho\left[\delta + \gamma\right] = 0 \tag{1.84}$$

$$- p_2 \frac{\partial F^{2j}}{\partial K_{2_j}} + \rho\left[\delta + \gamma\right] = 0 \tag{1.85}$$

And since $\dot{\lambda}_{1_j} = \dot{\lambda}_{2_j} = 0$ this means that, though a small change in $\dot{K}_{1_j}$ or $\dot{K}_{2_j}$ has an effect on the value of the Hamiltonian, since $\lambda_{1_j} = \lambda_{2_j} = \rho$ this means it will be *invariant* over the interval $[0, T]$. One feature of a genuinely dynamic element in the capital stock would be that λ_{1_j}, λ_{2_j} have non-constant time paths. For the pollution stock we shall see that this definitely is the case for many of the possible optimal trajectories.

Secondly, for the pollution stock the steady-state implies (since ε_{1_j} and ε_{2_j} are, from 1.77-8, functions of p_3)

$$\Sigma\left[\varepsilon_{1_j}(p_3) + \varepsilon_{2_j}(p_3)\right] - \alpha S = 0 \qquad (1.86)$$

so that

$$\left.\frac{\partial S}{\partial p_3}\right|_{\dot{S}=0} = \frac{\Sigma\left[\varepsilon_{1_j}' + \varepsilon_{2_j}'\right]}{\alpha} < 0 \qquad (1.87)$$

We assume, for simplicity, that

$$\left.\frac{\partial^2 S}{\partial p_3^2}\right|_{\dot{S}=0} = 0 \qquad (1.88)$$

Thirdly, the costate variable has the properties

$$p_3\left[\alpha + \gamma\right] - D' = 0 \qquad (1.89)$$

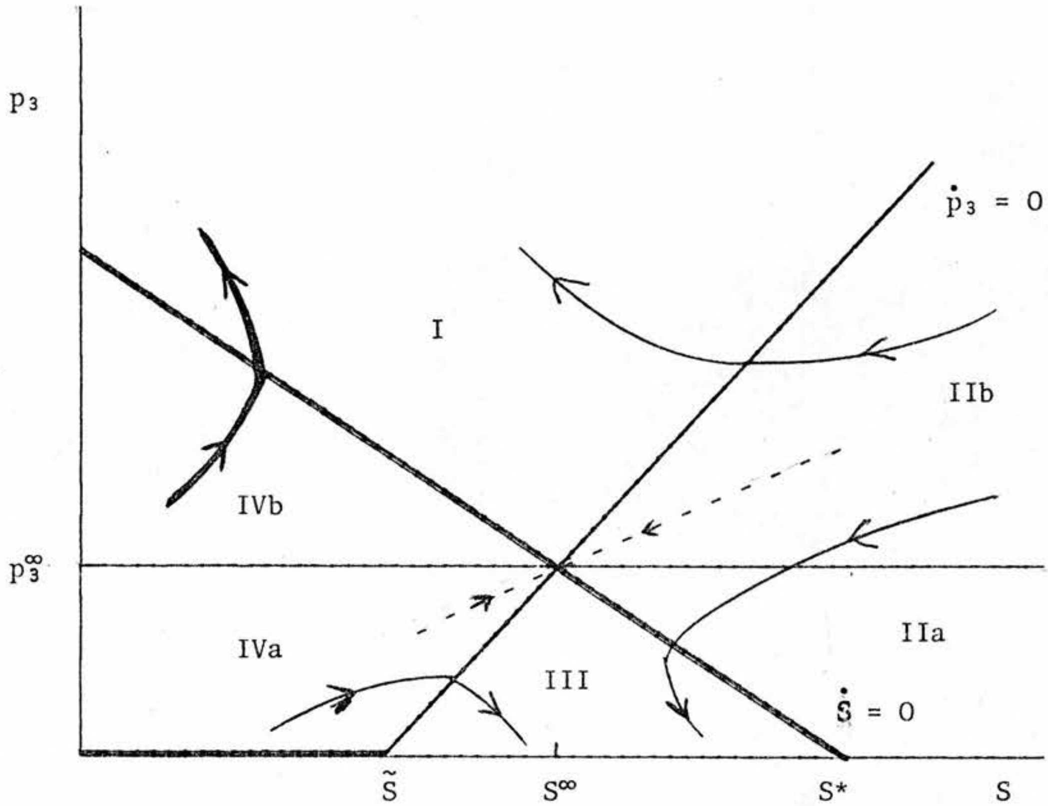
$$\left.\frac{\partial p_3}{\partial S}\right|_{\dot{p}_3=0} = \frac{D''}{\alpha + \gamma} = 0 \, , \quad S \leqslant \tilde{S} \qquad (1.90)$$

$$> 0 \, , \quad S > \tilde{S} \qquad (1.91)$$

$$\left.\frac{\partial^2 p_3}{\partial S^2}\right|_{\dot{p}=0} = 0 \qquad (1.92)$$

Thus we can construct the following Phase Diagram.

Figure 2



There are thus four main regions delineated by the lines of steady-state. The properties of $\dot{p}_3$, $\dot{S}$ and $\dot{E}$ in these regions are presented in the Table below.

TABLE 1.4

Regions of Steady State

	$\dot{p}_3$	$\dot{S}$	$\dot{E}$
I	+	-	-
II	-	-	+
III	-	+	+
IV	+	+	-

The horizontal line at p_3^∞ divides the phase space into subregions with further different properties. We now describe the various trajectories optimal under different assumptions regarding S_o , S_T , p_{3_o} , p_{3_T} .

Notice first of all from Table 4 that the pollution stock and total discharges may have opposite time rates of change. (For example, if the system is in region II discharge may be increasing over time but not at a sufficiently high rate to make $E > \alpha S$ true at any point in time; conversely, if the system is in region IV, though discharges are decreasing over time, the velocity is not large enough to ensure $E < \alpha S$ at any point in time.)

A second observation relates to the coincidence of the steady state line $\dot{p}_3 = 0$ with the S -axis for $S \leqslant \tilde{S}$. This reflects the fact that in this region marginal social damages are zero: $D'(S) = 0$. From 1.75

$$\dot{\eta} = \eta\left[\alpha + \gamma\right] , \quad S\varepsilon\left[0 , \tilde{S}\right] \qquad (1.93)$$

Hence

$$p_3 = be^{\left[\alpha + \gamma\right]t} , \; S\varepsilon\left[0 , \tilde{S}\right] \qquad (1.94)$$

where b is an arbitrary constant $(\geqslant 0)$ corresponding to the initial value of the shadow price of the pollution accumulation, p_3 . A once-and-for-all choice of $p_3(0)$ thus determines the behaviour of p_3 throughout the interval $[0 , \tilde{S}]$.

There are thus two possibilities:

(a) $p_3 = 0$, $S \varepsilon \left[0 , \tilde{S}\right]$.

Then $p_3 = 0$ all $S\varepsilon[0 , \tilde{S}]$ and $\dot{E} = 0$, implying that discharges are fixed at 'free good' levels $\overline{E}$ (to be determined by market prices, technology, etc.). Using the stock equation $\dot{S} = E - \alpha S$ we have

$$S(t) = \int_0^t \overline{E} e^{-\alpha[t-i]} di + S_o e^{-\alpha t} \qquad (1.95)$$

where

S_o is the inherited stock.

Integration yields

$$S(t) = \frac{\overline{E}}{\alpha}\left[1 - e^{-\alpha t}\right] + S_o e^{-\alpha t}$$

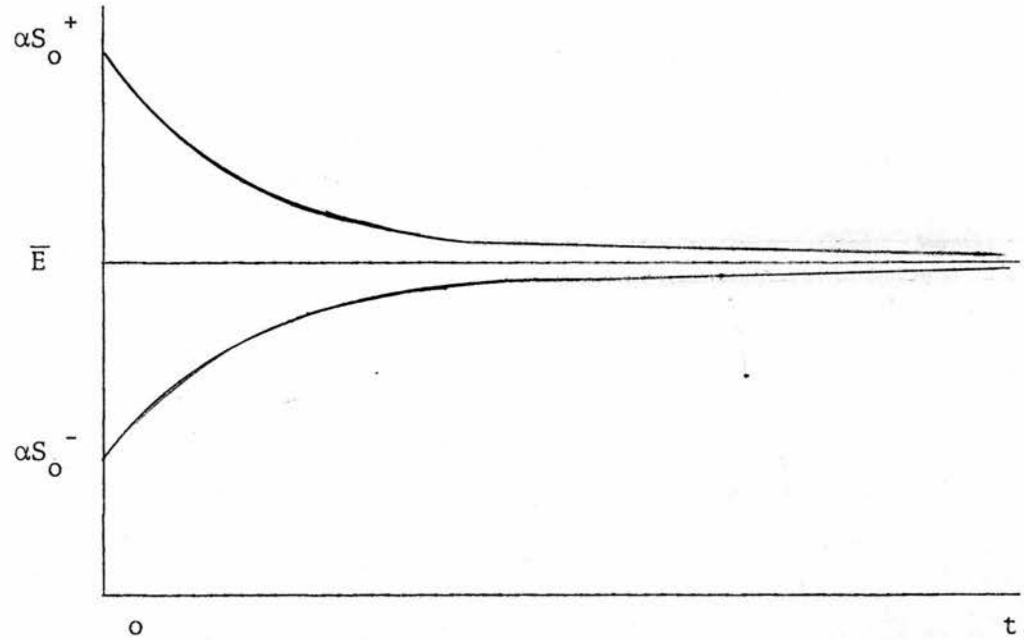
$$> 0 \text{ for } S_o > 0 . \qquad (1.96)$$

$$\dot{S}(t) = \left[\overline{E} - \alpha S_o\right] e^{-\alpha t} \gtreqless 0 \text{ as } \overline{E} \gtreqless S_o \qquad (1.97)$$

$$\ddot{S}(t) = -\alpha\left[\overline{E} - \alpha S_o\right] e^{-\alpha t} \lesseqgtr 0 \text{ as } \overline{E} \gtreqless S_o \qquad (1.98)$$

Thus S is a monotonic increasing or decreasing function over the interval $[0 , \tilde{S}]$, depending on the relation between the initial discharge rate and initial quantity of the inherited stock decayed. Diagrammatically:

Figure 2a



The instant, if such exists, at which the stock reaches $\tilde{S}$ is

$$t_m = - \left[\ell n\left(\frac{\overline{E} - \alpha\tilde{S}}{\overline{E} - \alpha S_o}\right)\right] \Big/ \alpha . \qquad (1.99)$$

If T is large then S tends asymptotically to $\overline{E}/\alpha > \overline{E}$ (since $0 < \alpha < 1$). The question therefore arises: If for large T, S tends to $\overline{E}/\alpha$, is this below or above $\tilde{S}$? Liapunov Stability Analysis provides the answer to this question and it turns out that $S\varepsilon[0 , \tilde{S}]$ *cannot* persist 'in the long run': it is a globally unstable solution. (This, moreover, is true regardless of the magnitude of the discount and decay rates (γ and α).)

The behaviour of the system in the region $[0, \tilde{S}]$ is particularly interesting for the light it casts on static optimisation mistakenly applied to a dynamic pollution phenomenon. Setting a constant tax rate $p_3 \geqslant 0$ under the erroneous assumption that this would produce a once-and-for-all optimum level of pollution, the Authority would find firms adjusting to long-run static optimum discharge levels at $t = 0$ and then remaining at that rate of discharge. The *stock* of pollution, however, does not remain static but increases (or, less probably, decreases) over time. A static stock level is in general *only* compatible with $\dot{p}_3 = 0$ if $S = E$ (i.e. $\alpha = 0$); all pollution would then be like noise, an instantaneous reverberation in the ambient medium with no further impact. The insidious machinations of substances like Caesium 231, Cadmium and Mercury do not however, conform to this romantic vista and pollution authorities must mount an eternal doomwatch to avoid their cumulative effects.

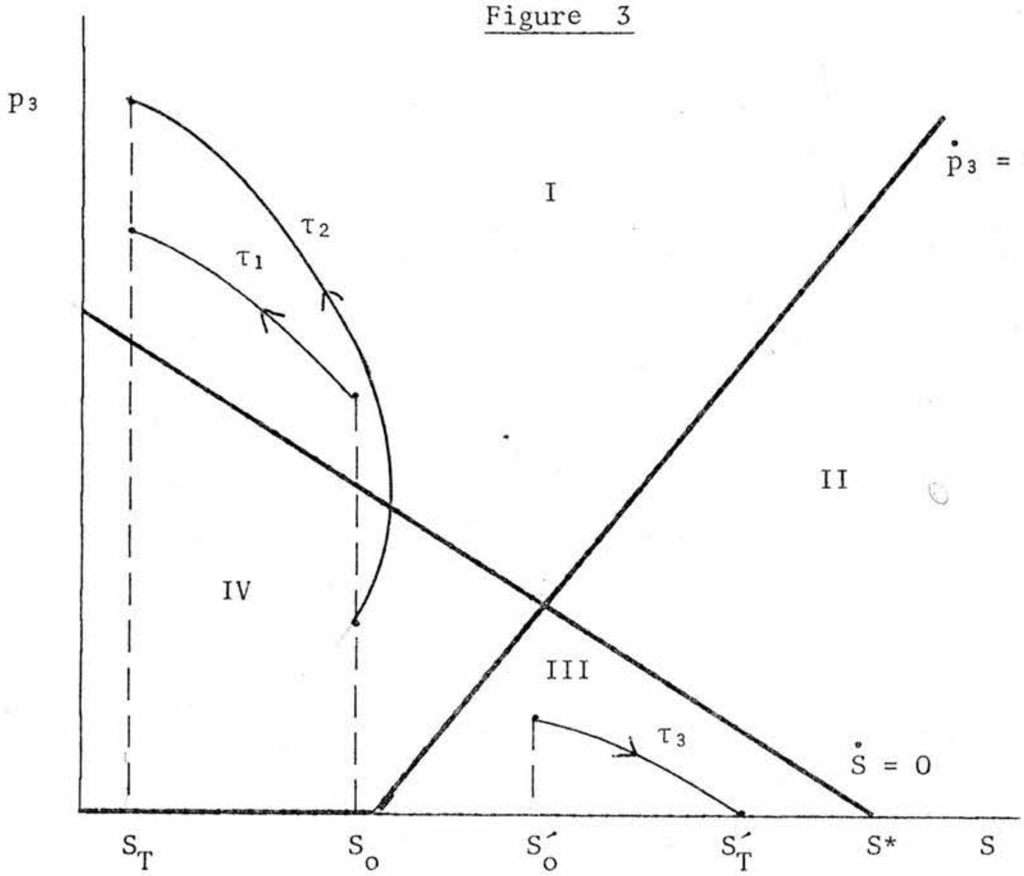
(b) $p_3(0) > 0$, $S\varepsilon[0, \tilde{S}]$

Then $\dot{E} < 0$ all $S\varepsilon[0, \tilde{S}]$, and from Table 1.4 $\dot{S} \gtrless 0$ as the trajectory falls in region IV or I. A positive shadow price says that there is a positive social gain attached to reducing the rate of pollution accumulation: thus *emissions* are reduced over time. However, the rate at which emissions decrease may or may not be sufficient to bring about a reduction in the total stock of pollution; 'in the long run' trajectories like τ_1, and τ_2 in Figure 3 must be nonoptimal, but ones like τ_1, in

Figure 4 may be optimal. For small discount rates and large T a final stock in region III will be globally stable.

The social optimum will, in general, be achieved at a higher level if we abandon the fixed end-point of the problem and allow the transversality condition to operate. This clearly demonstrates that optimally, starting in the region $[0\ ,\ \tilde{S}]$ the pollution stock cannot decrease over time.

Figure 3



Consider now the problem of selecting an optimal control in which the stock of pollution at the terminal instant of the planning period is unspecified but, rather than treating the final instant (T) as given we wish to regard it as endogenous to the optimisation problem. Thus our task is to determine an *optimal* T , say T* . It can be shown that, in addition to the transversality condition, such a procedure requires the value of the Hamiltonian at T* be zero. In the present model, then, we must have

$$\Sigma\pi_j(\cdot)_{T*} = D(S(T*)) \tag{1.100}$$

Where $\pi_j(\cdot)_{T*}$ is the jth firm's profit function evaluated at T* . Hence the *total* social benefit of production must equal the *total* social cost of the pollution stock it gives rise to at the terminal date. From an economic point of view this conclusion is obvious: if by expanding or contracting the planning period by a small amount the authorities could increase net benefits they would clearly have not reached a maximum of benefit; at the maximum, therefore, net benefits with respect to time must be zero. By means of transversality we may also conclude that

$$p_3(T*) = p_1\left[\frac{\partial F^{1j}}{\partial \gamma_{1_j}} + \frac{\partial F^{1j}}{\partial \varepsilon_{1_j}}\right]_{t=T*} = p_2\frac{\partial F^{2j}}{\partial \varepsilon_2}\bigg|_{t=T*} = 0 \tag{1.101}$$

Hence, at the optimal terminal date benefits to the firm from pollution in both production *and* recycling departments are at a maximum.

Substituting this result in 1.75 yields

$$\begin{aligned}\dot{p}_3(T*) &= -D'(S(T*)) \\ &= 0 \quad \text{for} \quad S(T*) \leqslant \tilde{S} \\ &< 0 \quad \text{for} \quad S(T*) > \tilde{S}\end{aligned} \tag{1.102}$$

The time rate of change of marginal production benefits from the pollution stock ($\dot{p}_3$) at the optimal terminal time exactly equals the (negative of) marginal social cost of pollution at that instant. Furthermore, if the terminal stock is smaller than the damage threshold ($S(T^*)\varepsilon[0\ ,\ \tilde{S}]$) then the time rate of change of pollution benefits to the firm is zero; if the terminal stock exceeds the threshold marginal benefits to the firm must be decreasing over time at the final date. From Figure 2, however, all paths in region IVa not tracing the line $\dot{p}_3 = 0$ must have increasing p_3 and S ; p_3 cannot start at a non-zero level, and either $p_3(0) = p_3(T^*) = 0$ and S moves along the S-axis from S_o to $S(T^*)$, or, the trajectory starts outside the region delimited by $[0\ ,\ \tilde{S}]$. Notice that for a trajectory commencing in $(\tilde{S}\ ,\ S^*]$ then $\dot{p}_3(T^*) \neq 0$ and $p_3(T^*) = 0$ imply a monotonically increasing pollution stock, the path being in region III (see τ_3 , Figure 3). Also, $S(T^*) \leqslant S^*$ always for an optimal arc.

The conclusions that may be drawn from the above analysis are, then, that for an *optimal* length planning period and/or terminal pollution stock it can never be desirable to reduce pollution, and a fortiori to reduce it to zero, if the initial stock is within or at the environmental threshold. This kind of policy can only 'become' optimal if the planners' hands are tied by time and/or the political necessity to pre-specify the final level of the pollution stock exogenously. Such a solution, as we have shown, can only be 'second-best'.

Ideally, then, both the length of the planning period and the final pollution stock should be endogenously determined from the equations of the control problem itself. Only in this way can a global maximum of net benefits be obtained. In such a context, the only obvious justification that could be claimed for not increasing pollution would be the presence of uncertainty about changes in the pollution stock in relation to industrial discharge. A discussion of that issue, however, is postponed to a later date.

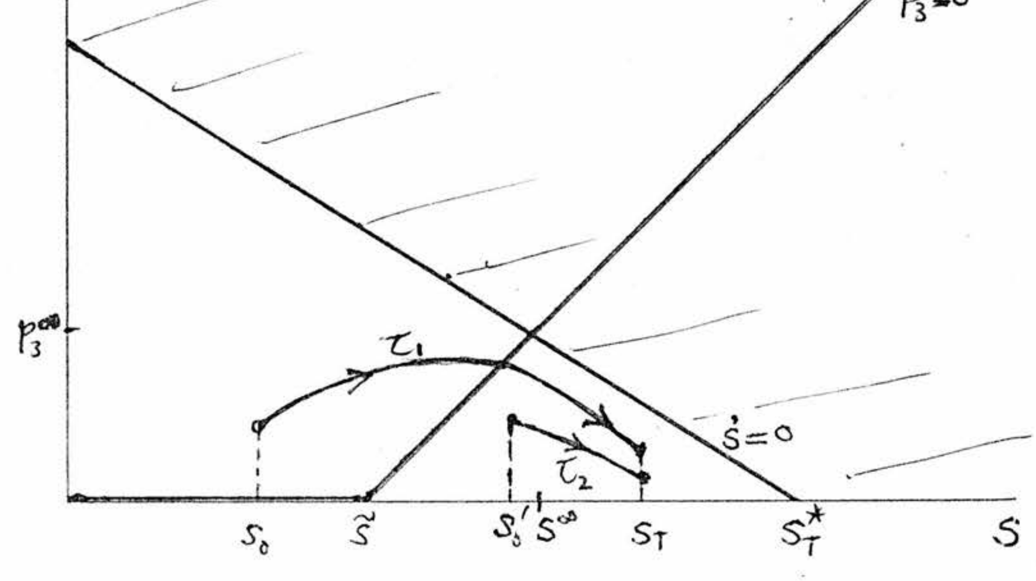
As mentioned by Førsund, if the horizon T is distant enough the optimal path is approximately that for an infinite horizon over the time-period in question. This possibility is represented in Figure 2 by the dotted arrows converging from above and below S^{∞} or $(S^{\infty}, p_3^{\infty})$. Such a trajectory cannot start in I, IIa, III or IVb, and the pollution stock is monotonically increasing or decreasing over $[0, T]$, depending on whether $S_0 \lesseqgtr S^{\infty}$.

If T is not large then several possibilities arise. We shall discuss three.

(i) $S_0 < S^{\infty} < S_T$

The constellation is depicted in Figure 4.

Figure 4



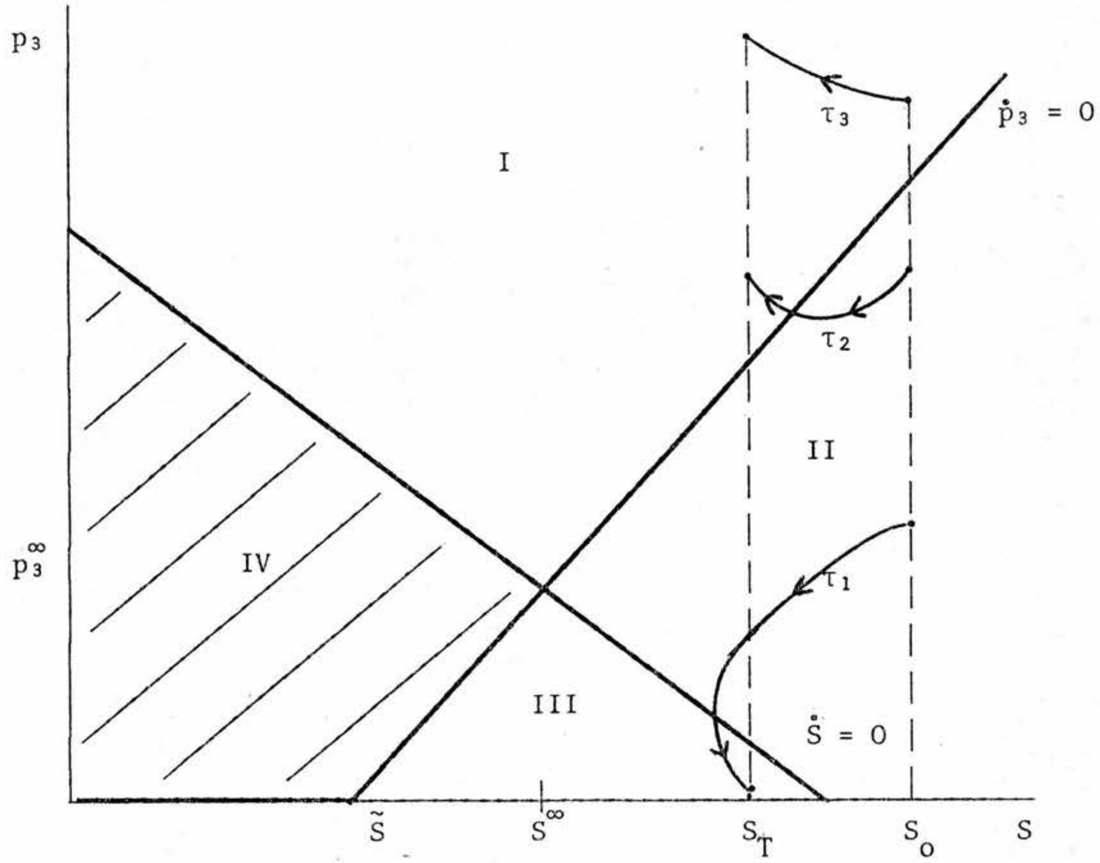
Shaded regions I and II are prohibited. The optimal trajectory may start at S_o or S_o' to terminate (its path monotonically increasing over time) at S_T. $S_T > S_T^*$ can never be optimal.

The two arcs τ_1 and τ_2 have of course quite different properties in p_3: in the second case p_3 is monotonically decreasing; in the first it rises, reaches a maximum, then falls to $p_3(T)$. From equation 1.82 this latter entails that the MRP of pollution in production first experiences a rise relative to MRP of recycling, reaches a maximum differential and then falls. Since prices are constant this can only be due to changes in the ratio of input magnitude ε_{1_j} and μ_{2_j} so that recycling becomes initially substituted for pollution discharge ($\frac{\varepsilon_{1_j}}{\mu_{2j}}$ decreases), the process then being reversed. A change in the ratio of pollution to recycling in each firm is of course perfectly compatible with increasing discharges in absolute terms.[24]

(ii) $S^{\infty} < S_T < S_o$

The constellation is delineated in Figure 5.

Figure 5

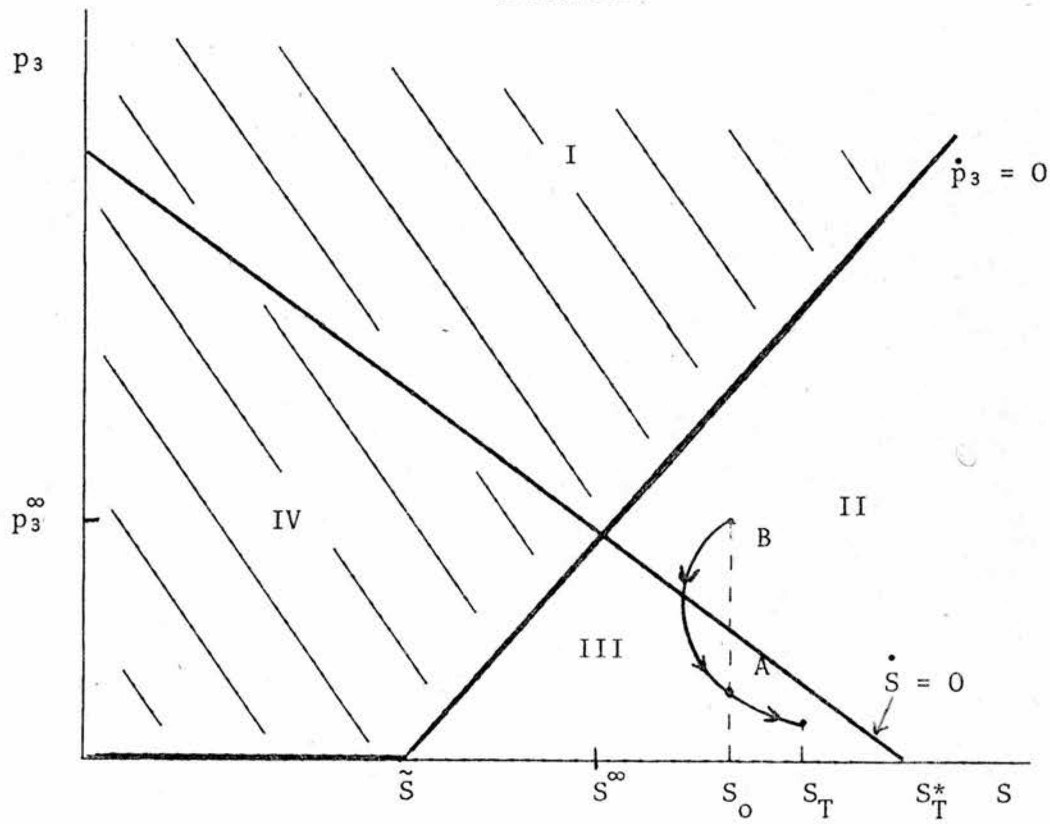


The trajectory can never start in region III and region IV is ruled out by assumption. Optimal arcs with S_o as in the figure may move from region II to III (arc τ_1) thus necessitating that the objective (S_T) is over-achieved before being achieved, or move from region II to region III in a monotonic attainment of the final pollution stock. The former path implies a (monotonic) decreasing marginal pollution benefit, the latter initially a decreasing, then an increasing pollution benefit. Hence τ_2 arises from the initial temporal substitution of pollution for recycling and then a reversal of this process. Finally, the optimal arc may reach the terminal stock from within region I (arc τ_3) again implying increasing marginal benefits to production.

(iii) $S^{\infty} < S_o < S_T$

The constellation is depicted in Figure 6.

Figure 6



Here the shaded regions I and IV are prohibited. Clearly it can never be optimal to have $S_T > S_T^*$. The optimal arc may start in either region II (point B) or region III (point A) to terminate in region III. In the former case as in (ii) above over-achievement of the objective final stock is apparent; the shadow price is monotonically declining in both situations.

1-5.5 Policy Conclusions for the Dynamic Model

We have seen that the Planning Authority may employ the dynamic shadow price p_3 to influence firms' polluting behaviour in such a way that the pollution stock proceeds along a predefined optimal trajectory. Whether pollution will be increasing (monotonically or nonmonotonically) depends on the specified initial and final conditions. If no final condition is specified then the supervention of the transversality condition implies $p_3(T) = 0$. This effectively rules out regions I and IVb (see Figure 2) for the optimal path and $\tilde{S} < S(T) < S^*$ results.

1-6 SUMMARY

In this chapter we have elucidated the concept of pollution in common parlance and historically; provided a classsification system for the different occurring varieties of pollution; and examined the problems associated with measuring associated costs and benefits. We have developed new theoretical structures both on a static and dynamic basis for an analytical examination of pollution phenomena, conceptualising discharged residuals as joint products and factors of production, and demonstrating both the comparative-static and dynamic amenability of pollution to fiscal intervention at a micro-economic level. Recycling and technological change as two main factors operating to change pollution magnitudes were also examined analytically, the former in a dynamic as well as static context. An important feature of the dynamic model here presented is the use of a social damage function with a threshold of perceived toxicity of pollution. It was shown that in a situation where society's initial pollution stock is less than this threshold it is socially irrational to set the final stock anywhere within this range since benefits are foregone to no obvious purpose. The optimal terminal stock of pollution should therefore be allowed to be determined endogenously to the system rather than be imposed from outside. Such a result may contradict a basic maxim of the environmentalist lobby, namely the overriding optimality of 'zero pollution'. This, however, will depend on the precise interpretation given to the term 'pollution'.

CHAPTER 1:

FOOTNOTES

1 Kings 2: 19-23. Quoted in Beckerman [1].

2 The literature on this is considerable, but Ehrlich and Ehrlich [4], and Commoner [5] are good examples of the line of thought.

3 The most notorious example of this was in Minnemata, Japan. For a short description of this disaster and other environmental characteristics of Mercury, see OECD [7].

4 See especially Stern [6], for a comprehensive documentation of recent research on air pollution.

5 But see OECD [8] and Ridker [9], PAU [10] for some useful attempts in this direction.

6 See Samuelson [2] and Buchanan [3] for the theory of public goods.

7 See D. Storey [11] for some U.K. water pollution data on this question.

8 See, e.g. Ridker [9] - who attempts to estimate marginal social costs and benefits of air pollution in America.

9 Except for output levels.

10 Pollution abatement, or negative pollution, is the output of an economic sector in this kind of model.

11 See section 1-5.

12 See Ayres and Kneese [13] and chapter 3 below for discussion.

13 A cost-minimiser under identical conditions could not, of course, optimise over pollution at all.

14 The more general case $Y = \emptyset(k, \ell)$ is mentioned by Ethridge.

15 The analogy is with the bus-driver who takes on the additional job of fare-collection: in Ethridge's theory the bus must always be *in motion* when the driver takes the money!

16 Waste paper collection in the U.K. in the past few years is a good example of just such a phenomenon.

17 Clearly if the processes produced generally quite different kinds of residuals and recycled commodities things could become extremely complicated.

F O O T N O T E S (cont.)

18 Recall that $\frac{\partial F^1(\cdot)}{\partial \varepsilon_1} = \frac{\partial F^1}{\partial \gamma_1} \frac{\partial \gamma_1}{\partial \varepsilon_1} = \frac{\partial F^1}{\partial \gamma_1}$.

19 In particular, if the recycling is for other than environmentally-orientated reasons the hypothesis could sometimes be violated.

20 Pearce [27] does not seem to be clear in his discussion of the environment's assimilative capacity whether he is referring to the relation between ε and αS or the relation between $\alpha(S)$ and S —— where α is a decreasing function of S. The distinction is, of course, crucial: if increasing S leads to $\alpha = 0$, then we have from 1.59 $\dot{S} = E$ and the stock increases over time for all $E > 0$; if $\alpha = 1$ (a 'pure flow' pollutant) then $S = E$ and the 'stock' is identical with the flow so that the time-dimension can be entirely neglected. Our analysis is restricted to the case where $0 < \alpha < 1$ and α = constant. The former assumption is generally more reasonable than Pearce's. The latter avoids problems of multiple equilibria.

21 In fact, we could imagine positive benefit being derived from low levels of some pollutants. E.g., SO_2 in low concentrations is a nutrient to crops.

22 This procedure is called "comparative dynamics". See Arrow and Kurz [28]..

CHAPTER 1:

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CHAPTER 2:

INPUT-OUTPUT ECONOMICS

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INPUT-OUTPUT ECONOMICS

2-1 INTRODUCTION

Chapter one provided an analysis of pollution at the micro or firm level based on traditional assumptions of continuous substitution possibilities amongst inputs and/or outputs. The planning authority's environmental problem was couched in terms of maximising an objective function defined over a set of such individual firms. Certain kinds of policy, however, have economy-wide implications and for this reason a *ceteris paribus* assumption is inappropriate; one planning authority's actions may create waves of influence that 'ramify into the farthest corners' of the economy. In such a situation we must move from partial to general equilibrium analysis, leaving the atomistic firm behind and replacing it with an aggregate of industrial establishments, the industry. To make this web of interactions empirically tractable it is necessary to invoke certain simplifying assumptions regarding production relationships among firms and final demand. These assumptions can yield the familiar input-output system of Leontief and his followers (see Leontief [11]), or an Activity Analysis model (see, e.g. Chenery and Clark [12]).

In this chapter we shall spend some time in analysing the formal characteristics of these two types of models, concentrating especially on the former. By way of justification for this apparent digression it should be sufficient to recall John Locke's concept of the philospher as underlabourer whose function is to clear away some of the 'dead wood' from the foreground of knowledge. Such analysis is a necessary prelude to the clear-sighted application of input-output techniques and in particular for their use in the study of ecologic-economic interactions — the subject of later chapters.

2-2 GENERAL FORM OF THE INPUT-OUTPUT SYSTEM WITH FIXED SET OF ACTIVITIES

Take an economy separable into n interdependent productive activities or sectors for each of which an additive measure of output is available. Interdependence amongst sectors implies that at least some sectors require output produced by other sectors in order to function as economic units. Sectors therefore produce in general a supply of a commodity for 'intermediate' usage (i.e. usage within the economic system) as well as for 'final' usage (i.e. consumption by households). The total output of any given commodity, say the ith, thus decomposes into quantities supplied to other productive units plus the amount supplied to households. Symbolically, if q_{ij} denotes the amount of the ith commodity absorbed by the jth industry, f_i the amount absorbed by the final demand sector, and q_i the total output of that commodity, we have:

$$\left.\begin{array}{l} q_{11} + q_{12} + \ldots + q_{1n} + f_1 = q_1 \\ q_{21} + q_{22} + \ldots + q_{2n} + f_2 = q_2 \\ \cdot \\ \cdot \\ \cdot \\ q_{n1} + q_{n2} + \ldots + q_{nn} + f_n = q_n \end{array}\right\} \qquad (2.1)$$

Let it be further postulated that the quantity of a commodity absorbed by an industry depends uniquely on the level of output of that industry and nothing else, i.e.:

$$q_{ij} = F_{ij}(q_j) \qquad (i,j = 1, \ldots n) \qquad (2.2)$$

where F_{ij} is a specific and determinate function of q_j. System 2.1 may then be rewritten as:

$$\left.\begin{array}{l} F_{11}(q_1) + F_{12}(q_2) + \ldots + F_{1n}(q_n) + f_1 = q_1 \\ F_{21}(q_1) + F_{22}(q_2) + \ldots + F_{2n}(q_n) + f_2 = q_2 \\ \cdot \\ \cdot \\ \cdot \\ F_{n1}(q_1) + F_{n2}(q_2) + \ldots + F_{nn}(q_n) + f_n = q_n \end{array}\right\} \qquad (2.3)$$

2-2.1 The Linear Hypothesis

Possibilities for the functional forms F_{ij} are virtually innumerable, and viewed from this perspective the scope of the input-output technique is clearly considerable. However, with the criterion of simplicity and operationality in mind it is equally obvious that in the estimation of such a system of functional relationships with limited quantities of data and in the context of a dynamic economy, gains in terms of functional accuracy (if possible at all) may be easily outweighed by losses in terms of temporal applicability. Theorists have for this reason largely concentrated on the simplest of functional forms, the linear relationship. Thus, in general 2.2 reduces to:

$$q_{ij} = a^*_{ij} + a_{ij}q_j \qquad (i,j = 1, \ldots n) \qquad (2.4)$$

in which

a^*_{ij}, and a_{ij} are constants.

2-3 THE LEONTIEF SYSTEM

2-3.1 Proportionality Assumption

It has become conventional in describing the analysis developed by Leontief to simplify the input-output relationships still further and drop the constant term a^*_{ij} of equation 2.4, thereby hypothesising the direct proportionality of the inputs absorbed by a given sector to the activity level of that sector:

$$q_{ij} = a_{ij}q_j \qquad (i,j = 1, \ldots n) \qquad (2.5)$$

One advantage accompanying this assumption is a set of very simple existence conditions for a solution to the system of equations 2.3.

2-3.2 Solution to the System

Adopting the assumption 2.5, we may write the system 2.3 as:

$$\left.\begin{array}{l} a_{11}q_1 + a_{12}q_2 + \ldots + a_{1n}q_n + f_1 = q_1 \\ a_{21}q_1 + a_{22}q_2 + \ldots + a_{2n}q_n + f_2 = q_2 \\ . \\ . \\ . \\ a_{n1}q_1 + a_{n2}q_2 + \ldots + a_{nn}q_n + f_n = q_n \end{array}\right\} \quad (2.6)$$

or, in matrix notation:

$$Aq + f = q \quad (2.7)$$

wherein

$$A = [a_{ij}].$$

An iterative solution to the system thus defined may be envisaged as follows (see Evans [6]).

Denoting our first estimate of the production level required by the ith sector to satisfy intermediate (endogenous) and final (exogenous) demand for its products by $q_i^{(1)}$ we have:

$$q_i^{(1)} = f_i + a_{i1}f_1 + a_{i2}f_2 + \ldots + a_{in}f_n \quad (2.8)$$

But the production of each such $q_i^{(1)}$ requires inputs from other sectors; hence a second estimate of the total output of the ith sector $q_i^{(2)}$, is needed:

$$q_i^{(2)} = f_i + a_{i1}q_1^{(1)} + a_{i2}q_2^{(1)} + \ldots + a_{in}q_n^{(1)} \quad \ldots\ldots (2.9)$$

However, by the same logic *these* inputs also require other commodities as inputs to *their* production, giving a third estimate of the output of the ith sector, $q_i^{(3)}$. In general, the kth such estimate may be written:

$$q_i^{(k)} = f_i + a_{i1}q_1^{(k-1)} + a_{i2}q_2^{(k-1)} + \ldots + a_{in}q_n^{(k-1)} \quad \ldots\ldots (2.10)$$

Since this equation is valid for *any and every* sector i, we can write, in matrix notation:

$$\left.\begin{aligned} q^{(1)} &= f + Af \\ q^{(2)} &= f + Af^{(1)} = f + Af + A^2f \\ &\cdot \\ &\cdot \\ &\cdot \\ q^{(k)} &= f + Af^{(k-1)} = f + Af + A^2f + \ldots + A^kf \end{aligned}\right\} (2.11)$$

This way of analysing the system brings out the fact that q, as the limiting term of the sequence $q^{(1)}, q^{(2)}, \ldots q^{(k)}$ is composed of a final demand element f, 'direct' inputs Af, and 'indirect' inputs given by $\sum_{k=2}^{\infty} A^kf$. This limit may or may not exist however. If it does then it must clearly be equivalent to finding the inverse of the matrix (I-A) and then postmultiplying by the final demand vector f; for,

$$(I-A)\ (I+A+A^2+\ldots+A^k) = I - A^{k+1} \qquad (2.12)$$

Now, if $\lim_{k\to\infty} A^{k+1} = 0$, 0 being the null matrix we have

$$(I-A)\sum_{k=o}^{\infty} A^k = I \qquad (\text{with } A^o=1) \tag{2.13}$$

or,

$$\sum_{k-o}^{\infty} A^k = (I-A)^{-1} \tag{2.14}$$

The conditions for $\lim_{k\to\infty} A^{k+1} = 0$ to hold good are simply

$$0 \leqslant a_{ij} < 1 \qquad (i,j = 1, \ldots n) \tag{2.15}$$

and

$$\sum_i a_{ij} < 1 \qquad (j = 1, \ldots n) \tag{2.16}$$

(For proof see Hadley [10], p.37, from which this analysis is taken.)

It should be carefully borne in mind nonetheless, that these conditions are merely necessary for us to be able to write the solution to the Leontief system *as a power series*, as in 2.11. Other methods of inversion are usually still open to us if these conditions are violated, and we shall in fact make good use of this point in later chapters.

Summarising the results of this Section in matrix notation, the solution to the Leontief model may be written, on the assumption that $(I-A)^{-1}$ exists, as:

$$q = (I-A)^{-1}f \tag{2.17}$$

2-3.3 Pricing of Commodities

Prices are the 'duals' or counterparts of quantities in economics. In the input-output system the concept of duality is defined in a very simple way. Rewriting equation 2.7 in the form

$$(I-A)q = f \tag{2.18}$$

the dual system is obtained by transposing the net coëfficients matrix, substituting for the endogenous quantity vector q an endogenous price vector p, and for the exogenous final demand vector f another exogenous vector y, of industry 'value-added' or 'primary' (unproduced) inputs.

This yields

$$(I-A)'p = (I-A')p = y \tag{2.19}$$

with

$$p = [p_j]$$

$$y = [y_i]$$

In 2.18 the problem was to solve the system for a vector of total outputs q, given a vector of final demand f. The dual of this problem is to derive a set of prices of industry/commodity outputs p, given the vector of primary inputs to industries/commodities, y. Conditions for the existence of a solution to the dual system are, in fact, identical with those for the primal. This follows directly from the fact that the transpose of the inverse of a matrix equals the inverse of its transpose. Thus, if $(I-A)^{-1}$ exists then so does $(I-A)^{-1'} = (I-A')^{-1}$. Solving 2.19 for p, we have:

$$p = (I-A')^{-1}y \tag{2.20}$$

An important feature of the Leontief system emerges on comparing equations 2.17 and 2.20; namely, that prices are independent of output. Entailed by this is the assumption of constant returns to scale since prices of inputs as well as outputs fall under this proposition of independence with respect to quantities. It also follows that an increase in demand is always met by an increase in output rather than a rise in price.

Whilst the statistical validity of the assumption of constancy in the A matrix with respect to small variations in q is amply corroborated by experience, over long periods and large variations the assumption is not inviolate. In other words, we may have for some element a_{ij} of A some relation such as

$$a_{ij} = a_{ij}(q) \tag{2.21}$$

holding good in these circumstances. In this event we thus create, through the mechanism of equation 2.20, a dependence of prices on outputs, i.e.

$$\frac{\partial p}{\partial q_i} = \frac{\partial}{\partial q_i}(I-A')^{-1}y \neq o \text{ , for some } i \tag{2.22}$$

Equation 2.20 has an analogous interpretation to 2.17 in terms of a matrix power series provided conditions 2.15 and 2.16 are valid. The price of the ith commodity is then conceived as composed of 'direct' factor inputs y, plus factor inputs into the commodities required to produce a unit of the industry's output, $A'y$, plus factor inputs into *those* inputs, A'^2y, and so on. The resulting matrix series is thus

$$\left.\begin{aligned} p &= y + A'y + A'^2y + \ . \ . \ . + A'^{(k-1)}y \\ &= (I+A'+A'^2+\ldots+A'^{(k-1)})y \end{aligned}\right\} \qquad (2.23)$$

2-3.4 The Leontief Accounting System and Alternatives

The interindustry accounting system employed by Leontief assumes a one-to-one correspondence between industries and commodities. Involved in this proposition is the conjunction of two component assumptions: that each industry produces only one commodity, and that each commodity is produced by only one industry. Prior manipulation of the raw data exhibiting multiple production by industries and commodities supplied by a diversity of sources is thus necessary if the one-to-one accounting framework is to be employed. In particular, subsidiary production of an industry must be allocated to other industries in accordance with the basic hypothesis of technological homogeneity of commodities produced in an industry, i.e., that there is a *unique* 'input structure' (set of input-output coefficients *re* the industry's production) obtaining for a given sector.

The requirement that commodities and industries be one-to-one is imposed by the need to invert the matrix (I-A). There are, however, considerable practical difficulties in dealing with the problem of subsidiary production. For example, detailed information on the input structure of industries is required if input-output tables are to be constructed effectively. Since procedures using such information have also not easily been reducible to computing algorithms, the delay arising from information collection is compounded by the laboriousness of computations. All this adds to the time-lag between the production of an input-output table and the year to which it 'ideally' applies. Consequently, the coefficients derived for the model are more liable to errors arising from price-quantity relationships (such as 2.22 above). It is therefore some advantage both practically and theoretically to develop an alternative basic accounting framework obviating the need for commodities and industries to be in one-to-one correspondence. The Stone System, described in the next section, fulfils this criterion and, as we shall see, provides considerable flexibility in the analysis of industry production functions and interindustry relationships.

2-4 THE STONE SYSTEM

2-4.1 Commodity-By-Industry Accounts

The accounting framework employed by Stone ([5] p.48) is schematically represented in the table below.

TABLE 2.1

Accounting Framework of Stone System

	Commodities	Industries	Final Demand	Totals
Commodities		$X = [x_{ij}]$	$f = [f_i]$	$q = [q_i]$
Industries	$M = [m_{jk}]$			$g = [g_j]$
Primary Inputs		$y = (y_j)$		
Totals	q'	g'		

Notation: Upper-case letters are used to represent matrices; lower-case letters, vectors; primes denote transposition. Untransposed vectors are column vectors. Subscripts

> i and k refer to commodities; j refers to industries. A piece of notation commonly employed in this section is the circumflex (^) over a vector indicating a matrix formed by diagonalising that vector.

Specific symbols in the accounting framework are defined as follows:

- X = the 'Absorption' matrix, showing the input of commodities into industries;
- M = the 'Make' matrix, giving the output of commodities by industries;
- f = a vector of final demand for commodity outputs;
- q,g = vectors of total commodity and industry outputs respectively;
- y = a vector of primary inputs into industries.

The remaining symbols are explained in context.

The first 'row' of the accounting framework expresses the identity of the total quantity of each commodity produced with the sum of intermediate and final demands for that commodity:

$$q = X1 + f \tag{2.24}$$

The second 'row' of the accounts shows that the sum of the quantities of all commodities produced by an industry constitutes that industry's aggregate output:

$$g = M1 \tag{2.25}$$

Commodity and industry outputs can also, due to the symmetrical nature of the accounting system, be got by summing over the first and second 'columns' of the table; in the case of commodities (first 'column') this enunciates the identity of the outputs of a commodity from all industries with the total domestic production of that commodity:

$$q = M'1 \tag{2.26}$$

and in the case of industries (second 'column'), it demonstrates the decomposition of industry output into intermediate and primary *in*puts:

$$g = X'1 + y \tag{2.27}$$

For completeness we may add two more basic accounting identities. Firstly, the sum of industry and commodity total outputs are equal:

$$1'g = 1'q \tag{2.28}$$

And, combining 2.24 and 2.27 we can infer

$$1'X1 + 1'f = 1'X'1 + 1'y \tag{2.29}$$

but we have

$$1'X1 = 1'X'1 \tag{2.30}$$

therefore,

$$1'f = 1'y. \tag{2.31}$$

In other words, total consumers' expenditure on commodities equals national income.

2-4.2 Structural Matrices: Simple Forms

From the basic identities outlined in the previous section three simple structural matrices may be defined.

A matrix of *market shares,* D, of dimensions industry x commodity:

$$D = [d_{ji}] = M\hat{q}^{-1} \tag{2.32}$$

and a matrix of *commodity mixes* of industries, C, order commodity x industry:

$$C = [c_{ij}] = M'\hat{g}^{-1} \tag{2.33}$$

exhibit respectively the distribution of commodity outputs amongst industries and the proportions in which commodities combine to make up industry outputs. Finally, a matrix of industry input coefficients, B:

$$B = [b_{ij}] = X\hat{g}^{-1} \tag{2.34}$$

shows the amounts of various commodities required by industries *per unit* of output.

2-4.3 General Forms of Input-Output Model

By defining an industry-into-commodity transformation matrix $T = [t_{jk}]$ the basic identities of 2-4.1 yield expressions for total commodity and industry outputs as functions of an exogenous bill of final demand:

$$\begin{aligned} q &= Xl + f && \text{from 2.24} \\ &= Bg + f && \text{using 2.34} \\ &= BTq + f && \text{since } g = Tq \text{ by definition} \\ &= A_Tq + f && \text{with } A_T = BT \\ &= (I - A_T)^{-1}f && (2.35) \end{aligned}$$

Similarly,

$$\begin{aligned} g &= Tq \\ &= TBg + Tf \\ &= E_Tg + e_T && \text{with } E_T = TB, e_T = Tf \\ &= (I - E_T)^{-1}e_T && (2.36) \end{aligned}$$

Thus, the hypothesis of multiproduct industries results in two input-output systems counterpart to Leontief's model of equation 2.17.

2-4.4 Technology Verbally Defined

Equations 2.35 and 2.36 will yield projections of commodity and industry outputs respectively from a given vector of final demand once A_T and E_T are known or estimated. Commodity and industry technology assumptions, first introduced by Stone, Bacharach and Bates [3], go some way towards determining the choice of the transformation matrix T and the behaviour of A_T, B, and E_T over time.

The *input structure* of a commodity k with respect to any commodity i is the amount of i required per unit of output of k. We now define:

> *Commodity Technology Hypothesis: (C.T.H.)*
> the input structure of any commodity is *the same* for all industries. In other words, the input structure of every commodity is independent of industries.
>
> *Industry Technology Hypothesis: (I.T.H.)*
> the input structure of commodities is determined by their industry of origin. That is, all commodities produced in a *given* industry have *the same* input structure.

These two verbal definitions are essentially those given in Stone, Bacharach and Bates ([3] pp.13-14). However, the concepts were never couched in a precise mathematical form in that work and consequently the relationship between these underlying formulae and the A_T and E_T matrices was never made sufficiently clear. Confusions in the literature are ubiquitous and so a rigorous derivation of these formulae and the structural matrices to which they relate is worthwhile.

2-4.5 Analytics of Technology Assumptions

Let α_{ikj} be the input of the ith commodity per unit of output of the kth commodity when the latter is produced in the jth industry. If x_{ij} is the total absorption of the ith commodity by the jth industry, we have

$$x_{ij} = \sum_k \alpha_{ikj} m_{kj} \qquad \text{all } i,j \qquad (2.37)$$

with m_{kj} the 'make' of the kth commodity by the jth industry. Furthermore, denoting the input coefficient of the jth industry with respect to the ith commodity by b_{ij}, and the industry's output by g_j, we can write down an expression for the former as the weighted average of the input coefficients of commodities absorbing i:

$$\begin{aligned} b_{ij} &= x_{ij}/g_j && \text{all } i,j && (2.38) \\ &= \sum_k \alpha_{ikj} m_{kj} / \sum_k m_{kj} && \text{using 2.37 and recalling } g_j = \sum_k m_{kj} \\ &= \sum_k \alpha_{ikj} c_{kj} && && (2.39) \end{aligned}$$

with $c_{kj} = m_{kj}/\sum_k m_{kj}$ the proportion of the kth commodity in the jth industry's output.

Industry technology asserts that, for any i,j:

$$\alpha_{i1j} = \alpha_{i2j} = \ldots\ldots\ldots\ldots = \alpha_{imj} \qquad (2.40)$$

where m is the number of commodities.

Hence from 2.39 industry technology implies, for any i,j:

$$b_{ij} = \alpha_{ikj} \qquad \text{all } k \qquad (2.41)$$

contrary to U.N.([5] Sect. 3.86), where it is asserted that A_D is the input coefficients matrix under industry technology.

Commodity technology takes the form of an assertion that, for any i,k:

$$\alpha_{ik1} = \alpha_{ik2} = \ldots\ldots\ldots\ldots = \alpha_{ikn} \qquad (2.42)$$

n being the number of industries.

Specifically, for any i,k:

$$a_{ik} = \alpha_{ikj} \qquad \text{all } j \qquad (2.43)$$

where $[a_{ik}] = A_T$.

2-4.6 Constant Technical Conditions

Constant technical conditions mean the input structures of commodities are constant. From 2.39 and 2.41 it is clear that a static technical situation is neither necessary nor sufficient for fixity of $B = [b_{ij}]$ unless the industry technology hypothesis is adopted. Similarly, constant technical conditions only become necessary and sufficient for the fixity of A_T in the context of the commodity technology assumption (as can be seen from 2.43).[1] Equation 2.41 also elucidates the fact that an unchanging technology under the dispensation of the industry technology hypothesis is not sufficient to facilitate prediction, since A_T and T may vary jointly over time. Hence for projection purposes it is necessary to initially project T. Under a commodity technology, however, the hypothesis that A_T is constant is adequate for the employment of the model in prediction of commodity outputs.[2]

2-4.7 Form of Transformation Matrix

It might seem that the precise pattern of the transformation matrix has not yet been considered. However, it is easy to see that the *commodity* technology assumption implies restrictions on the form for matrix T. For, substituting 2.43 in 2.39 we have

$$b_{ij} = \sum_k a_{ik} c_{kj} \tag{2.44}$$

Or, in matrix notation

$$B = A_T C \tag{2.45}$$

with $C = [c_{ij}]$, the matrix of 'commodity mixes' of industries.

The general solution to this system is:

$$A_T = BZ(CZ)^{-1} \tag{2.46}$$

where Z is some (arbitrary) nxm matrix satisfying the condition

$$\rho(Z) = m \leqslant n \tag{2.47}$$

Specifically, if $Z = C^{-1}$ we get

$$A_T = BC^{-1} \tag{2.48}$$

But it is *not* necessary to have C square to obtain a solution in terms of B and C; for, putting $Z = C'$:

$$A_T = BC'(CC')^{-1} \tag{2.49}$$

This result contradicts the assertion made by Stone, Bacharach and Bates ([3] p.22) and U.N.([5] p.50) that $\bar{C}$ must be square for a C.T. solution. What is in fact necessary is that the number of industries should not be *less* than the number of commodities. Under this dispensation we have

$$T = Z(CZ)^{-1} \tag{2.50}$$

In terms of simple known matrices we may choose $Z = D$ a matrix of 'market shares' of industries, yielding

$$T = D(CD)^{-1} \tag{2.51}$$

It can readily be seen that the solution

$$A_D = BD \tag{2.52}$$

is impopssible in normal circumstances under commodity technology. For then we have

$$BD = BZ(CZ)^{-1} \tag{2.53}$$

and so

$$D = Z(CZ)^{-1} \quad \text{if} \quad \rho(B) = n \leqslant m \tag{2.54}$$

To be compatible with 2.47 this requires $n = m$; but even if this condition is satisfied we still need

$$(DC-I)Z = 0 \tag{2.55}$$

Now, Z is non-null *IFF*

$$|DC-I| = 0 \tag{2.56}$$

but in normal circumstances this will *not* be the case. Since in practice we *choose* Z as a non-null matrix, this contradicts our assumptions. Q.E.D.

No counterparts to equations 2.46 and 2.50 can be derived simply from the industry technology hypothesis. Substitution of 2.41 into 2.39 merely reproduces an identity which has no implication for the form of the transformation matrix T:

$$b_{ij} = \sum_k b_{ij} c_{kj} = b_{ij} \Sigma c_{kj} = b_{ij} \quad \text{since} \quad \Sigma c_{kj} = 1 \qquad \ldots\ldots (2.57)$$

Therefore an industry technology hypothesis in the context of the input-output model leaves the form of the industry-into-commodity transformation matrix entirely undetermined, apart from a possible, but not necessary, requirement that it should satisfy certain accounting conventions. From this it is evident that 2.48 is perfectly compatible with an industry technology assumption, contrary to assertions in the C.S.O.'s 1968 publication ([2] p.25) and Armstrong ([1] Sect.3) where it is suggested that 2.48 is a sufficient condition for identifying a *commodity* technology assumption underpinning the model. Gigantes' work seems to display an implicit awareness of this fact in his discussion of various different possible forms of output structure under industry technology ([4] pp.272-280) but nowhere is the principle enunciated.

Since the commodity technology assumption does not entail constancy of C it is clearly not implicit in such models that commodity outputs (of any kind) are proportional to industry outputs. An assumption of the fixity of C *can* of course be made under the dispensation of a commodity technology - as also under an industry technology employing $A_C = BC^{-1}$, for example. However, one of the factors contributing to the superior plausibility of the commodity technology assumption is precisely the possibility of variations in the output structure matrix, for there is no reason to assume that principal and ordinary subsidiary products[3] will be produced in fixed proportions to one another in the composition of each industry's output. This virtue is therefore negated by the hypothesis suggested in the U.N. System ([5] p.49).

2-4.8 Bi-Products

Bi-products, being produced in fixed proportions to principal products (we do not here consider bi-products to subsidiary commodities) possess an output structure that is easily accommodated. Treatment of the input structure of bi-products does, however, present conceptual difficulties, only part of which are peculiar to the input-output system. Two alternative treatments seem to be available, but in neither case is it generally valid to assign their input structure to an industry technology as is commonly believed ([3] p.13; [4] p.284; [5] p.50; [1] Sect. 6).

(i) In the event that it is possible to distinguish a *separate* input of some given product into both the principal and bi-product(s) then the input structure of these products is determined only by examining particular empirical cases; there is no *a priori* presumption in favour of an industry technology for the bi-product(s). For example, suppose the production of a certain organic chemical produces sulphuric acid as bi-product. The quantity of fuel 'absorbed' by the principal product per unit of output may be taken as the total input into the process to produce that one unit excepting the input of fuel into (say) the motors required to siphon off the acid. No obvious presumption exists to the effect that the latter coefficient will be identical with the input structure of the industry.

(ii) If there is no obvious rule by which the per unit absorption of a commodity by a process can be allocated amongst the principal and bi-products then the following situation obtains.

Let the process in question result in the output of r products $k = 1, \ldots r(\leq m)$. Then the input structures of such products re the ith commodity sum to the input coefficient of the process (as measured in value terms):

$$\alpha_{i1p} + \alpha_{i2p} + \ldots + \alpha_{ijp} \ldots + \alpha_{irp} = \sum_k \alpha_{ikp} = \sigma_{ipj} \qquad (2.58)$$

α_{ijp} being the input coefficient of the principal product of the jth industry and σ_{ipj} the input coefficient of the pth process in that industry. Hence there exist only $r - 1$ degrees of freedom in assigning the values of the α_{ikp}'s, the rth coefficient being determined by the constraint represented in 2.58. We are not therefore free to assign (say) the principal product to a commodity technology and the $r - 1$ bi-products to an industry (or commodity) technology: only $r - 1$ products (of any kind) may be so assigned. In the case where there is just one bi-product to the principal product the implication is that if the principal is assigned to a commodity technology (say) then it is not in general possible to allocate an industry (or commodity) technology to its bi-product. A fact of importance in case (ii) is that, within the process constraints, the input structure of bi-products is not an empirical question since we are, ex hyp., arbitrarily specifying the input structure in question. (This does not seem to be generally appreciated since writers discuss the application of technology in this case as if it were an hypothesis that could be confirmed or disconfirmed by experience.) Therefore, as regards prediction, the choice of input structure (given that it satisfies the mentioned constraint) is quite immaterial and cannot affect the result. The only sensible criterion here is that of *convenience*.

2-4.9 Hybridisation

Flexibility may be introduced into models 2.35 and 2.36 above by the device of *hybridisation*. Logically, this procedure may be split into two parts (though not completely independent, as has been indicated); namely, the choice of input and the choice of output assumptions for specific subsets of commodities.

9.1 Hybrid output structure assumptions describe certain alternative industry and hybrid (or mixed) technology output matrices. In this subsection we shall confine ourselves to industry technology models, reserving mixed technology for subsection 9.2. We saw in section 2-4.7 that the industry technology hypothesis, unlike the commodity technology hypothesis, leaves the form of output structure indeterminate. Opened up by this eventuality, then, is the possibility of defining subsets of commodity outputs by industries according to *different* output criteria. Suppose two such subsets are envisaged. Then it is possible to represent this hypothesis by a corresponding subdivision of the Make matrix,

$$M = M_1 + M_2 \tag{2.59}$$

M_1 representing those outputs in the Make matrix to be given (say) a commodity mix assumption, and M_2 those to be assigned (say) a market share hypothesis. By definition of the matrices in equation 2.59 we have corresponding subdivisions of commodity and industry outputs:

$$q = q_1 + q_2 = M_1'1 + M_2'1 \tag{2.60}$$

$$g = g_1 + g_2 = M_1 1 + M_2 1 \tag{2.61}$$

(the on-line 1's representing summation vectors).

In Gigantes' [4] models (his equations 8 and 9) bi-products are considered to be produced in fixed proportions to industry outputs, whilst principal and ordinary subsidiary products are assigned fixed market shares. However, the first of these assumptions, viz. that

$$C_2 = M_2'\hat{g}^{-1} \tag{2.62}$$

where

M_2 is a Make matrix of industry outputs of bi-products,

and

g is a vector of (total) industry outputs,

is constant seems, *prima facie,* less plausible than the alternative

$$C_2 = M_2'\hat{g}_1^{-1} \tag{2.63}$$

with

M_2 as in 2.62,

and

g_1 a vector of industry outputs of prinipal products.

For, if bi-products are proportional to principal production, as is clearly the case, it follows that C_2 (2.62) will be stable if and only if the ratio of principal to non-bi-product subsidiary production is constant. Nevertheless, C_2 (2.63) seems to possess a radical disadvantage of requiring to satisfy the conditions for non-singularity if it is to function as a component in a hybrid output matrix, whereas C_2 (2.62) does not.[4] (In fact a malaise of this kind appears to afflict most matrices C_2 defined in terms of 'subdivisions' of g (as g_1, g_2 above).)

Gigantes' second assumption, that D_1 defined by

$$D_1 = M_1 \hat{q}_1^{-1} \tag{2.64}$$

where

M_1 is a Make matrix of outputs of principal and ordinary subsidiary products,

and

q_1 is a vector of outputs of commodities produced as the principal and secondary products of industries,

is stable, notably does not express the outputs in the rows of M_1 as a proportion of q, the outputs of products produced under *all* modes of production. That is, Gigantes' assumption must be distinguished from

$$D_1 = M_1 \hat{q}^{-1} \tag{2.65}$$

with q a vector of outputs of commodities. It might in fact be argued that it is artificial to distinguish a 'market' for principal and ordinary secondary products since the description under which they are classified merely serves to identify certain analytical characteristics of their production and does not ascribe to them a separate demand function.[5]

U.N. ([5] equation 3.12) suggests the employment of a commodity mix matrix C_1 such that

$$C_1 = M_1 \hat{g}_1^{-1} \tag{2.66}$$

in which

M_1 is the Make matrix of industry outputs of principal and ordinary subsidiary products,

and

g_1 is a vector of industry outputs of such products.

The rationale behind this is that M_1 should contain outputs to be treated on a commodity technology assumption. Nonsingularity of C_1 is here again a precondition of its use, but in normal circumstances the matrix will clearly satisfy this condition. However, Stone's apparent belief that entailed in the commodity assumption is the fixity of commodity mixes wrongly leads him to hypothesise constancy of C_1. Whilst this proposition may easily be waived in using the model, more substantial objections may be raised against Stone's other output assumption, the market share matrix D_2:

$$D_2 = M_1 \hat{q}^{-1} \qquad (2.67)$$

in which

M_1 is a Make matrix of bi-products (etc.),

and

q is defined in 2.65 above.

This hypothesis seems to have emanated from an association of market shares with industry technology. The proposition being disproved, such a choice appears quite arbitrary: there is no reason to assume the market shares of bi-products are stable.

9.2 The Stone Model just discussed has already implicitly introduced the concept of mixed input technology, but a more systematic development of the idea from the foundations laid in sections 2-4.4, 2-4.5, and 2-4.6 is requisite.

Hybrid technology assumptions take the form of a subdivision of commodity outputs by industries such that, for any given industry, say the jth, certain of its products are considered to have an input structure specific to the industry and the remainder to be determined independently of the industry. Thus, as we saw with hybrid output structure assumption, mixed input technology results in a division of the Make matrix into two components M_1 and M_2 satisfying the equation

$$M = M_1 + M_2 \tag{2.68}$$

where M_1 now contains commodity technology outputs (say and M_2 industry technology outputs (say). The procedure is tantamount to representing the absorption of any commodity (say the ith) into a given (jth) industry as the sum of its absorption into production of commodities in class χ (commodity technology) and those in class η (industry technology). Thus we have

$$x_{ij} = x_{ij}^{\chi} + x_{ij}^{\eta} \tag{2.69}$$

Therefore, a corresponding subdivision of the industry input coefficients matrix exists:

$$\begin{aligned} b_{ij} &= b_{ij}^{\chi} + b_{ij}^{\eta} \\ &= (x_{ij}^{\chi} + x_{ij}^{\eta})/g_j \end{aligned} \tag{2.70}$$

expanding 2.70 we have[6]

$$\begin{aligned} b_{ij}^{\chi} &= x_{ij}^{\chi}/\sum_k m_{kj} \\ &= \sum_{k\varepsilon\chi} \alpha_{ikj} m_{kj}/\sum_k m_{kj} \\ &= \sum_{k\varepsilon\chi} \alpha_{ikj} c_{kj} \end{aligned} \tag{2.71}$$

and similarly,

$$b_{ij}^{\eta} = \sum_{k\varepsilon\eta} \alpha_{ikj} c_{kj} \tag{2.72}$$

Commodity and industry technology assumptions applied respectively to equations 2.71 and 2.72 yield

$$b_{ij}^{\chi} = \sum_{k\varepsilon\chi} a_{ik} c_{kj} \qquad \text{(com. tech.)} \tag{2.73}$$

which in matrix notation becomes

$$B_1 = A_1C_1 \tag{2.74}$$

and

$$b^{\eta}_{ij} = b_{ij}\sum_{k\varepsilon\eta} c_{kj} \qquad \text{(ind. tech.)} \tag{2.75}$$

or,

$$B_2 = B\hat{C}_2^{1} \tag{2.76}$$

wherein:

$$B_1 = [b^{\chi}_{ij}], \qquad B_2 = [b^{\eta}_{ij}]$$

and

$$C_1 = [c^{\chi}_{kj}], \qquad C_2 = [c^{\eta}_{kj}], \qquad A_1 = [a^{\chi}_{ik}]$$

From 2.74, then, a static technical situation implies constancy of A_1; and from 2.76. constancy of B. Thus an industry technology assumption for merely a subset of commodity outputs in the model entails the constancy of the *whole* B matrix (B_2 may vary, depending on the behaviour of C_2, and B_1 depending on C_1). The commodity-balance equation may now be written as

$$A_1q_1 + B_2g + f = q \tag{2.77}$$

in which q_1 is the vector of commodity outputs assigned to a commodity technology.[7] It is evident that to solve for commodity outputs, say, some form of transformation matrix from industry outputs is required. Call this matrix H. Such a matrix is easy to derive utilising the hybrid output assumptions discussed in the previous section. Employing equations 2.62 and 2.64 (i.e. Gigantes' assumptions) we have

$$q = (I - A_1(I + C_1 C_1^{\wedge}1^{-1}H - CH))^{-1}f$$

$$= (I - BR)^{-1}f \tag{2.78}$$

$$g = Hq \tag{2.79}$$

with

$$R = (C_1^{\wedge}1)C_1^{-1}(I + C_1(C_1^{\wedge}1)^{-1}H - CH) \tag{2.80}$$

$$H = (I + D_1C_2 - C_2^{\wedge}1)^{-1}D_1 \tag{2.81}$$

If instead of 2.64 we use 2.65 the result becomes:

$$q = (I - BS)^{-1}f \tag{2.82}$$

$$g = Hq \tag{2.83}$$

with

$$S = (C_1^{\wedge}1)C_1^{-1}(I - D_2^{\wedge}1) + (I - C_1^{\wedge}1)H \tag{2.84}$$

To project commodity outputs by means of equations 2.78 and 2.82 we need to assume $A_R = BR$ and $A_S = BS$ constant respectively. Pragmatically this implies R and S must be constant over time, since B is stable.[8] Theorists have so far assumed this constancy is a consequence of D_1, D_2, C_1, and C_2 being stable. However we have seen there is little theoretical basis for such a supposition; likewise, there is no obvious reason to assume these matrices *vary* in such a way as to hold R or S constant. There is thus little *a priori* justification for choosing a mixed rather than a simple commodity technology. The problems associated with process constraints of section 2-4.8 lend further force to this argument. In the last resort the issue is settleable only by recourse to purely statistical testing of the various alternative assumptions, but at present data is inadequate for this purpose.

2-4.10 Pricing of Outputs

In section 4.96 of the S.N.A. [5], Stone employs two identities to derive an expression for primary inputs into commodities, given, *inter alia,* the datum of primary inputs into industries. His equations are:

$$g = y + \hat{g}B'1 \tag{2.85}$$

and

$$q = v + \hat{q}A_T'1 \tag{2.86}$$

where

- y is a vector of primary inputs into industries;
- v is an (unknown) vector of primary inputs into commodities.

Equation 2.85 has plain enough economic sense: it states that industry outputs decompose into primary and intermediate inputs. Equation 2.86 would seem to affirm that commodity outputs breakdown into primary inputs into commodities (the solution for which is to be derived) and intermediate inputs into commodities. But it will be perfectly clear from the analysis of technology of sections 4 and 5 that the expression

$$\hat{q}A_T'1$$

can only be interpreted as a vector of commodity inputs into commodities on the assumption that A_T represents the input structure of commodities. This, however, is the commodity technology assumption with the implication that we have $A_T = A_C = BZ(CZ)^{-1}$. Under an industry technology A_T represents the weighted sum of the input coefficients of industries; or, which is the same thing in this case, the weighted sum of the input structure of commodities. Stone's analysis does not, therefore, apply to models based on industry, and by implication, hybrid, technology.

In place of 2.86 we write the basic identity:

$$q_i = \sum_{kj}\sum \alpha_{ikj} m_{kj} + v_i \tag{2.87}$$

showing that the output of the ith commodity consists of primary inputs plus intermediate commodity inputs. Industry technology applied to this equation yields:

$$q_i = \sum_j b_{ij} g_j + v_i. \tag{2.88}$$

or, in matrix notation

$$q = Bg + v \tag{2.89}$$

But from 2.35

$$q = Bg + f$$

Hence

$$f = v \tag{2.90}$$

In other words, commodity value-added under I.T. is identically equal to final demand for that commodity.

We now proceed to derive the equations of the general solution for commodity value-added under a commodity technology assumption. The solutions obtained by Stone will then appear as special cases subsumed under this general formulation.

Employing a commodity-into-industry transformation matrix T^* in connexion with equation 2.86 we have

$$\begin{aligned} g &= T^*q \\ &= T^*v + T^*\hat{q}T'B'1 \\ &= y + \hat{g}B'1 \end{aligned} \tag{2.91}$$

therefore,

$$\begin{aligned} T^*v &= y + \hat{g}B'1 - T^*\hat{q}T'B'1 \\ &= y + [\hat{g} - T^*\hat{q}T']B'1 \end{aligned} \tag{2.92}$$

Now, the solutions obtained by Stone are:

$$y = Dv \tag{2.93}$$

and

$$y = C^{-1}v \tag{2.94}$$

but referring back to 2.92 we observe that

$$y = T^*v \quad \text{IFF} \quad [\hat{g} - T^*\hat{q}T']B'1 = 0 \tag{2.95}$$

and since $B'1 \neq 0$, and $b_{ij} \geqslant 0$, this necessitates

$$\hat{g} = T^*\hat{q}T' \tag{2.96}$$

if proposition 2.95 is to be valid. In the case that $T^* = C^{-1}$ and $T = D$, 2.96 becomes

$$\hat{g} = C^{-1}\hat{q}D' \tag{2.97}$$

which is true by virtue of the identities defining C and D. It is evident, however, that 2.92 admits of many solutions for which 2.96 may be violated, and it is not necessary to presuppose squareness of C for a commodity technology solution to the system to be possible.

We are now in a position to obtain the price formulae of our systems.

Commodity value-added coefficients, u, may be written:

$$u = \hat{q}^{-1}v \tag{2.98}$$

Combine this formula with a (known) vector of industry value-added coefficients, n, given by

$$n = \hat{g}^{-1}y \tag{2.99}$$

and we can derive an expression for the prices of commodities. There are two sets of price equations corresponding respectively to commodity and industry technology assumptions. The price of the kth commodity under a commodity technology may be written as

$$p_{kj} = \sum_i \alpha_{ikj} p_{ij} + u_{kj} \tag{2.100}$$

or

$$p_k = \sum_i a_{ik} p_i + u_k \tag{2.101}$$

where

a_{ik} is an element of $A_C = BZ(CZ)^{-1}$

p_{kj} is the price of the kth commodity as produced by the jth industry;

and

u_{kj} is the primary input coefficient for the kth commodity in the jth industry.

The corresponding industry technology price equation is:

$$p_{kj} = \sum_i \alpha_{ikj} p_{ij} + \mu_{kj} = \sum_i b_{ij} p_{ij} + \mu_{kj}$$

$$(k = 1, \ldots m; \; j = 1, \ldots n) \tag{2.102}$$

We note here that since I.T. implies

$$\mu_{1j} = \mu_{2j} = \ldots = \mu_{mj} = \mu_j \text{ (say)} \tag{2.103}$$

one may by reasoning along similar lines to those of section 5 deduce that

$$n_j = \mu_j \tag{2.104}$$

Comparing now equations 2.101 and 2.102 we can see that commodity technology yields a set of m unique prices for commodities, whereas industry technology, by making the input structure of commodities dependent on their industry of origin, results in n sets of m commodity prices, one set for each industry. In fact, however, we have:

$$\left.\begin{array}{l} p_{1j} = \sum_{i=1}^{m} b_{ij}p_{ij} + n_j \\ p_{2j} = \sum_{i=1}^{m} b_{ij}p_{ij} + n_j \\ \cdot \\ \cdot \\ \cdot \\ p_{mj} = \sum_{i=1}^{m} b_{ij}p_{ij} + n_j \qquad (j = 1, \ldots n) \end{array}\right\} \qquad (2.105)$$

so that

$$p_{1j} = p_{2j} = \ldots = p_{mj} = p_j \text{ (say)} \qquad (2.106)$$

$$(j = 1, \ldots n)$$

In other words, though there are n sets of commodity prices under industry technology, for any *given* industry the prices of all commodities are *identical*.

With this result in mind we may rewrite 2.105 as

$$\left.\begin{array}{l} p_1 = p_1\Sigma b_{i1} + n_1 \\ p_2 = p_2\Sigma b_{i2} + n_2 \\ \cdot \\ \cdot \\ \cdot \\ p_n = p_n\Sigma b_{in} + n_n \end{array}\right\} \qquad (2.107)$$

In matrix notation equations 2.101 and 2.107 may then be written:

$$p = A_c'p + u \tag{2.108}$$

and

$$p = \hat{B'1}p + n \tag{2.109}$$

with

$$p = [p_k]; \qquad n = [n_k]; \qquad u = [u_k]$$

Equations 2.108 and 2.109 give solutions of the form:

$$p = (I - A_c)^{-1}u \tag{2.110}$$

and

$$p = (I - \hat{B'1})^{-1}n \tag{2.111}$$

It is of interest to note that no writer on the subject of commodity-by-industry models has yet derived the equations for commodity prices presented in equation 2.109 above or noted the necessary distinction between pricing systems for different technology-based input-output models implicit in these equations.

Constant technical conditions under the commodity technology hypothesis imply constancy of A_C and u ; under the industry hypothesis of B and n . Apart from the relative complexity of pricing in an industry technology system which is an additional (practical) consideration against the use of this kind of assumption, an implication of equation 2.109 is a greater order of complexity for hybrid technology-based models.

2-4.11 Conclusions on Technology Assumptions

Our main conclusion is that simple commodity technology models still stand as the most viable forms of the Stone input-output system, and are quite amenable to the form of disaggregation implied by a rectangular Commodity Mix matrix. The lack of obviously plausible criteria for a subdivision of the Make matrix in accordance with technology and output structure assumptions impugns the predictive usefulness of current forms of hybrid accounting. The failure of theorists to appreciate the need for a set of commodity prices for each industry under the I.T. assumption also means that hybridisation must be reconsidered from the pragmatic aspect of complexity in pricing it implies, for there are non-unique prices for the I.T. subset of products.

2-4.12 The Open Economy

Let

$$Y = [y_{ij}] \tag{2.112}$$

be a commodity-by-industry matrix of (intermediate) imports. It has the same dimensions as the domestic absorption matrix X . Define an imports coefficients matrix B_I :

$$B_I = Y\hat{g}^{-1} \tag{2.113}$$

In order to determine the absorption of commodities imported by commodities home-produced we employ the familiar industry-by-commodity transformation matrix T :

$$A_I = B_I T \tag{2.114}$$

with A_I a commodity-by-commodity input-output matrix the ijth element of which shows the per unit absorption of the ith imported commodity into the jth domestic commodity output. Utilising the transformation $g = Tq$ we can write down an expression for the direct and indirect requirements of industry for imported commodities:

$$A_I q = A_I(I-A)^{-1}f \qquad (2.115)$$

Adding in imports absorbed directly into final demand, f^I, we get a vector i of total commodity imports by the economy:

$$i = A_I q + f^I = A_I(I-A)^{-1}f + f^I \qquad (2.116)$$

It is important to note that the specific form of T does *not* commit us to commodity or industry technology assumptions (contrary to CSO [2], p.26). Of course an implicit 'technology' assumption of some kind is involved in assuming constancy of either B_I or A_I; but since we assume Y has the *same* dimensions and commodity/industry set as X, entailed by this is the proposition that the breakdown of commodity per unit absorption of commodities by way of *source* (domestic or foreign) is independent of the input structure of domestically produced commodities; a market shares hypothesis $(T = D)$, say, for A_I cannot therefore in any way be related to domestic input-structural assumptions.

2-5 ACTIVITY ANALYSIS MODELS

2-5.1 Introduction: Nature of Linear Programming

The fundamental feature of all models of this type is a function to be optimised (an 'objective function') linear in a set of variables, and a set of linear constraints on the levels of these variables, including non-negativity constraints. The constraints may consist of equalities, inequalities or both. Inequalities can be converted to equalities by the introduction of 'slack' or 'surplus' variables in the constraints and objective function. A characteristic of the constraint set is that there are more variables than equations (when the former is expressed in equation form); thus implying, if the constraint set is consistent, that a number of alternative solutions exist. That solution(s) is chosen which maximises/minimises the value of the objective function.

A set of solutions, called 'basic' solutions, to the equation system thus defined may be obtained by setting all but m of the variables to zero. Clearly there are $^{n}C_{m}$ such solutions. We are generally interested only in those solutions satisfying the non-negativity constraints on the variables, so a subset from the class of basic solutions must be selected. The elements of this set are called basic *feasible* solutions. It can be shown that the optimal value of the objective function is located at one of the extreme points of the convex set of basic feasible solutions, and that in such a solution at most m variables (m being the number of equations or 'basic' variables, i.e. variables in the basis) need be different from zero. Related to the original or 'primal' programming problem is another, secondary, or 'dual' problem. The variables, parameters, and optimal solution to this latter problem have a specific relation to the primal problem which make the dual very interesting when the analysis is given an economic interpretation. We shall see that the relationship between primal and dual problems is somewhat more complicated in linear programming than in the Leontief System. Before discussing these features of programming in more detail it is necessary to transfer into symbolic notation.

The general form of a linear programming problem is:

$$\text{Minimise } C = c'x \tag{2.117}$$

Subject to

$$Ax \geqslant b, \quad x \geqslant 0$$

with dual:

$$\text{Maximise } V = b'p$$

Subject to

$$A'p \leqslant c, \quad p \geqslant 0 \tag{2.118}$$

where capital letters represent matrices, lower case letters vectors and primes denote transposition. c, b and A are the parameters of the system; x and p are vectors of primal and dual variables respectively. If we denote the optimal value of the primal objective function C by C^*, and that of the dual function V by V^*, then it can be demonstrated that $C^* = V^*$ (see, e.g. Lancaster [9]). Furthermore, subject merely to the condition that the original basis remains *feasible* (i.e. the basic variables remain non-negative) it can be shown that for any alternative primal constraint vector, b, the same basis is also optimal for the new program. Subject to the feasibility restriction on variation in the constraint vector, the change in the value of the objective function, ΔC^*, is specified by

$$\Delta C^* = p^* \Delta b$$

where p^* is the optimal dual vector. Hence:

$$\frac{\Delta C^*}{\Delta b_i} = p_i^* \tag{2.119}$$

with

Δb_i the change in the ith primal constraint

p_i^* the value of the ith dual variable.

Put in the language of the differential calculus this becomes

$$\frac{\partial C^*}{\partial b_i} = p_i^*$$

Subject to

$$x \geqslant 0 \tag{2.120}$$

2-5.2 Activity Analysis

Activity analysis is an application of the technique of linear programming to economic problems. Suppose a linear model of production and consumption with s activities a_j, m outputs x_j, ($m > s$), constituting m constraints on the levels of those activities. Each commodity here, as in the Leontief System, plays a dual role: both as input and as output. Each activity is represented symbolically by a (column) vector of input-output coefficients to the activity; in the case of production these are net inputs/outputs per unit of the activity. In the notation of the Stone commodity x industry accounting system the derivation of the set of activity vectors may be explained as follows.

Let M' be the transpose of the Make matrix;

X represent, as before, the Absorption matrix.

Then a matrix of net inputs/outputs of commodities *re* industries is defined by the equation

$$N = M' - X \tag{2.121}$$

being of order commodity x industry.

A vector of industry production levels, g, is given by

$$g = M1 \tag{2.122}$$

The set of production activities $A_E = (a_1^E, \ldots a_n^E)$ can now be derived by postmultiplying the matrix N by the inverse of the diagonal matrix $\hat{g}$:

$$A_E = (a_1^E, \ldots a_n^E) = N\hat{g}^{-1} \tag{2.123}$$

Thus, the typical element of A_E , a_{ij}^E , shows the net input/output of the ith commodity *re* the jth production activity (or *industry*), operating at unit level. Net outputs have positive, net inputs negative, signs attached.

An assumption of constancy in A_E clearly does not imply fixity in either C, the matrix of commodity mixes of industries, or B, the matrix of industry input structures, as defined in the Stone System. For, we can write equation 2.123 in the form:

$$A_E = C - B \tag{2.124}$$

where

$C = M'\hat{g}^{-1}$ is a matrix of commodity mixes,

and

$B = X\hat{g}^{-1}$ is a matrix of industry input coefficients.

Thus we subject the elements of C and B to a constraint of the form

$$\text{const.} = a_{ij}^{E} = c_{ij} - b_{ij} \tag{2.125}$$

but, within the bounds of this constraint the c_{ij} and b_{ij} are free to vary. It cannot therefore be maintained that in the activity analysis model "the proportions in which an industry absorbs different intermediate products are independent of the proportions in which that industry is producing different commodities" is erroneous. Equation 2.123 above shows that for a given commodity (i), because the difference between these proportions is assumed constant, any increase in the proportion in which the industry produces that commodity must be met by a corresponding increase in its proportional absorption of it. But the relation between the proportion in which an industry produces a commodity k and absorbs a *different* commodity i, is left *undetermined* in the activity analysis model. This is not, however, to assert that the activity analysis model implies these sets of quantities are *independent*. Put in other words, because the activity analysis model does not actually imply (say)

$$b_{ij} = F_{ij}(c_{1j}, \ldots c_{mj}), \quad \text{all } j \tag{2.126}$$

this is not tantamount to assuming no such relation exists; it merely implies that either (1) the functions F_{ij} are unknown, or (2) it is unnecessary to specify them for the purposes of applying the model (just as we do not have to project C and B in the C.T. model to project q, even though C and B may be functionally related).

The above analysis shows, therefore, that the concept of technical conditions in the activity analysis model here developed is in important respects different from the notions employed in the Leontief and Stone Systems discussed in sections 2-3 and 2-4.

The remaining (s - n) activities (which exclude 'slack' and 'surplus' activities) are consumption sectors. They are denoted by a matrix $E_E = (e^E_{n+1}, \ldots e^E_s)$. If $C = [c_{ij}]$ now represents a matrix of consumption patterns by sectors (c_{ij} showing the consumption of the ith commodity by the jth consumption sector) then the activity levels corresponding to these sectors can be defined by a vector k such that

$$k = C'1 \tag{2.127}$$

and the set of consumption activities is then defined from the relation

$$E_E = (e^E_{n+1}, \ldots e^E_s) = C\hat{k}^{-1} \tag{2.128}$$

with k a diagonal matrix formed from the elements of k . Equation 2.128 shows that the activity level of the jth consumption sector is simply defined as the sum of the quantities of all the commodities it consumes. The elements of E_E are all 0.

We now proceed to describe and interpret two simple types of economic activity analysis models.

Suppose there is no constraint on the quantity of natural resources used by the economy, or pollutants produced by it; this provision can be thought of in terms of 'infinite' constraints on the levels of these inputs and outputs which are 'ineffective' simply because they are not finite. The quantity of labour available in the projection period may, however, be realistically considered strictly limited. Two primal objective functions may be envisaged:

(i) Maximise consumption by households subject to technical conditions and a constraint on labour supply.

(ii) Minimise the national cost of supplying a *specified* vector of final demand for economic commodities, subject to technical conditions and a given supply of labour.

Symbolically, these two models may be written:

$$\text{(i)} \quad \left.\begin{array}{l} \text{Maximise } F = k_f \\ \text{Subject to} \\ \quad A_E g + E_E k + c k_f \geqslant 0 \\ \quad m' g \geqslant b_L \\ \quad g,\ x,\ k_f \geqslant 0 \end{array}\right\} \qquad (2.129)$$

where

- k_f is the activity level of the criterion sector, viz., consumption by households;
- c is a vector of activity coefficients for k_f;
- k is a vector of activity levels for the remaining consumption sectors;
- m is a vector of 'direct' labour input coefficients *re* production activities $(m_j \leqslant 0)$;
- b_L is the total quantity of labour available $(b_L \leqslant 0)$.

The remaining symbols have been defined earlier in this section.

(ii)
$$\left.\begin{array}{l} \text{Minimise } S = (s', 0')x \\ \\ \text{Subject to} \\ \\ \quad A_E g + E_E k \geqslant b_E \\ \\ \qquad m'g \geqslant b_L \\ \\ \qquad g, k \geqslant 0 \end{array}\right\} \qquad (2.130)$$

where

- S is national cost;
- s is a vector of prices or 'direct' costs of production activities;
- 0 is a null vector
- b_E is a vector of minimum levels of final demand for economic commodities.

All other symbols have already been defined.

Dual problems to models (i) and (ii) seek respectively to minimise resource costs (in terms of primary inputs) and to maximise the value of net national output. Writing primals (i) and (ii) in the form:

$$\left.\begin{array}{l} \text{(i)} \quad \text{Maximise } F = (0', 0', 1')x \\[2ex] \quad \text{Subject to} \\[2ex] \quad \left[\begin{array}{c|c|c} A_E & E_E & c \\ \hline m' & 0' & 0' \end{array}\right] x \geqslant \left[\begin{array}{c} 0 \\ \hline b_L \end{array}\right] \\[2ex] \quad x \geqslant 0 \end{array}\right\} \qquad (2.131)$$

$$\left.\begin{array}{l} \text{(ii)} \quad \text{Minimise } S = (s', 0')x \\[2ex] \quad \text{Subject to} \\[2ex] \quad \left[\begin{array}{c|c} A_E & E_E \\ \hline m' & 0' \end{array}\right] x \geqslant \left[\begin{array}{c} b_E \\ \hline b_L \end{array}\right] \\[2ex] \quad x \geqslant 0 \end{array}\right\} \qquad (2.132)$$

the corresponding duals (iD) and (iiD) are seen to be:

$$\left.\begin{array}{l} \text{(iD)} \quad \text{Minimise } R = (0', b_L)p \\[2ex] \quad \text{Subject to} \\[2ex] \quad \left[\begin{array}{c|c} A_E' & m \\ \hline E_E' & 0 \\ \hline c' & 0 \end{array}\right] p \leqslant \left[\begin{array}{c} 0 \\ \hline 0 \\ \hline 1 \end{array}\right] \\[2ex] \quad p \geqslant 0 \end{array}\right\} \qquad (2.133)$$

$$\left.\begin{array}{l}\text{(iiD)}\quad \text{Maximise } V = (b_E', b_L)p \\ \\ \quad\quad \text{Subject to} \\ \\ \quad\quad \left[\begin{array}{c|c} A_E' & m \\ \hline E_E' & 0' \end{array}\right] p \leqslant \left[\begin{array}{c} s \\ \hline 0 \end{array}\right] \\ \\ \quad\quad\quad\quad p \geqslant 0 \end{array}\right\} \qquad (2.134)$$

The first 'row' of dual (iD) has typical element

$$\pi_j = \sum_i a_{ij}^E p_i - m_j p_L \leqslant 0 \qquad (2.135)$$

which indicates that profits on the jth activity (π_j) must be *non-positive*. In the event that they are negative, the activity obviously operates at zero level, for it can be shown (see Lancaster [9] that

$$x_j^* = 0 \quad \text{whenever} \quad \sum_i a_{ij}^E p_i - m_j p_L < 0 \qquad (2.136)$$

where

x_j^* is the optimal solution value of the jth primal activity.

In this model there is only *one* shadow price, p_L, which may be interpreted as the rate of change of national consumption with respect to variations of labour supply:

$$p_L = \frac{\partial F}{\partial b_L} \qquad (2.137)$$

Because of the positive signs attached to consumption *activity* (not *inputs into* consumption) this derivative will have a positive sign implying that an increase in available labour increases household consumption. The first 'row' of the dual (iiD) has typical element

$$\pi_j = \sum_i a^E_{ij} p_i - m_j p_L \leqslant s_j \tag{2.138}$$

which asserts that the profits of the jth activity should not exceed some preassigned value s_j. Hence in this program profits may be positive. Shadow prices for (iiD) indicate the rate of change of national resource cost with respect to variations in specified minimum levels of final demand for commodities and labour supply:

$$p_i = \frac{\partial S}{\partial b_i} \qquad (i = 1, \ldots m; \; m+1) \tag{2.139}$$

2-5.3 The Open Economy

It is assumed that imports into intermediate and final activities are proportional to *domestic* activity levels. The maximisation of domestic final consumption is then made subject to the additional constraint that the total volume of imports (into intermediate *plus* final usage) is equal to the total volume of exports. Symbolically, if A^I represents the (intermediate) imports coefficients matrix, Y the matrix of intermediate import flows, E^I the final demand imports coefficients matrix, F^I the corresponding flow, and c^X the vector of export activity coefficients, we write

$$(A^I g + E^I k)1 = c^X 1 \tag{2.140}$$

the typical element of c^X denoting the input of commodity i into the export activity per unit of its total level of operation. Models i and ii above thus include now an export activity among the final demand sectors and optimisation becomes subject to the further constraint 2.140.

2-6. MATHEMATICAL APPENDIX

Proof of a Proposition in Section 2-4.9:

"If bi-products are proportional to principal production it follows that C_2 (2.62) is stable if and only if the proportion of principal to non-bi-product production is constant."

Let c^2_{ij} represent the typical element of C_2 (2.62), thus showing the proportion of the ith bi-product in the total output of industry j; then we have

$$c^2_{ij} = m^2_{ij}/(\sum_i m^1_{ij} + \sum_i m^2_{ij}) \qquad (1)$$

where m^1_{ij} denotes the output by industry j of product i produced as non-bi-product,

m^2_{ij} represents the output of bi-product i from industry j.

Suppose the output of the ith bi-product can be written as

$$m^2_{ij} = \delta_{ij} m_{jj} \qquad (2)$$

with m_{jj} the output of the principal products by industry j , and

δ_{ij} a factor of proportionality.

We can therefore rewrite 1 as

$$c^2_{ij} = \delta_{ij} m_{jj}/(\sum_i m^1_{ij} + \sum_i \delta_{ij} m_{jj}) \qquad (3)$$

Clearly if $\sum_i m^1_{ij}/m_{jj}$ = constant over the projection period then c^2_{ij} will be constant; and vice versa.

Proof of Proposition 2.82:

We have:

$$A_1 q_1 + B_2 g + f = q \tag{1}$$

$$g = Hq \tag{2}$$

By assumption,

$$D_2 = M_2 \hat{q}^{-1} \tag{3}$$

$$D_2 \hat{q} = M_2 \quad \text{and} \quad \hat{q} D_2' = M_2'$$

whence

$$\hat{q} D_2' 1 = q_2$$

or

$$\widehat{D_2'1}\, q = q_2 \tag{4}$$

Therefore, by substitution in 1:

$$A_1 (q - \widehat{D_2'1}\, q) + B_2 H q + f = q \tag{5}$$

Now, recalling that $B = B_1 + B_2$ and $A_1 = B_1 C_1^{-1}$ we have from this last equation:

$$(B - B_2) C_1^{-1} (I - \widehat{D_2'1}) q + B_2 H q + f = q \tag{6}$$

However, $B_2 = B\widehat{C_2'1}$; thus

$$B[(I - \widehat{C_2'1}) C_1^{-1} (I - \widehat{D_2'1}) + \widehat{C_2'1} H] q + f = q$$

and so,

$$q = [I - B[(I - \widehat{C_2'1}) C_1^{-1} (I - \widehat{D_2'1}) + \widehat{C_2'1} H]]^{-1} f \tag{7}$$

But this can be simplified further by recalling that

$$I - C_2^{\hat{!}}1 = C_1^{\hat{!}}1;$$

hence

$$q = [I - B[C_1^{\hat{!}}1C_1{}^{-1}(I - D_2^{\hat{!}}1) + (I - C_1^{\hat{!}}1)H]]^{-1}f$$

which is proposition 2.82. Q.E.D.

Proof of Gigantes' equation:

$$q = A_1[I + C_1(C_1^{\hat{\dagger}}1)^{-1}H - CH]q + f \tag{1}$$

(Although Gigantes must have provided a proof of this proposition when submitting his original paper this has not, to my knowledge, been published.)

$$g = Hq \qquad \text{by definition of } H \tag{2}$$

$$g_2 = C_2^{\hat{\dagger}}1g \tag{3}$$

$$A_1q_2 + Bg_2 + f = q \qquad \text{since } B_2g = Bg_2 \tag{4}$$

$$q_1 = q - q_2 = q - C_2g \tag{5}$$

Substituting 3 and 5 into 4:

$$A_1(q - C_2g) + BC_2^{\hat{\dagger}}1g + f = q \tag{6}$$

and

$$A_1(I - C_2H)q + BC_2^{\hat{\dagger}}1Hq + f = q \qquad \text{from 2} \tag{7}$$

But

$$C_2^{\hat{\dagger}}1 = I - C_1^{\hat{\dagger}}1 \tag{8}$$

Hence

$$BC_2^{\hat{\dagger}}1Hq = (B - BC_1^{\hat{\dagger}}1)Hq \qquad \text{and so 7 becomes:}$$

$$A_1(I - C_2H)q + (B - BC_1^{\hat{\dagger}}1)Hq + f = q \tag{9}$$

Now, $A_1C_1(C_1^{\hat{\dagger}}1)^{-1} = B$; thus 9 may be written:

$$[A_1(I - C_2H) + A_1C_1(C_1^{\hat{\dagger}}1)^{-1}(I - C_1^{\hat{\dagger}}1)H]q + f = q \qquad \ldots\ldots \tag{10}$$

or

$$A_1[I - C_2H + C_1[(\widehat{C_1'1})^{-1} - I]H]q + f = q \qquad (11)$$

Therefore,

$$A_1[I + C_1(\widehat{C_1'1})^{-1}H - C_1H - C_2H]q + f = q \qquad (12)$$

whence:

$$A_1[I + C_1(\widehat{C_1'1})^{-1}H - CH]q + f = q$$

Q.E.D.

CHAPTER 2:

FOOTNOTES

1. This definition of constant technical conditions is corroborated in the literature (See: [3] pp.27-30; [4] p.272).

2. Gigantes [4], equations 19 and 20 (which latter, by the way, should read: $g = (I-DAD^{-1})^{-1}De$, in his notation), shows that the output structure of commodity technology models for projecting industry outputs must be projected (or held constant) if the models are to be useful. This is not the case when commodity outputs are projected.

3. Definition: The *principal product* of an industry is that product of which, in value terms, it produces most. A simple case is where there are equal numbers of commodities and industries ($n = m$), and commodities are defined as industry principal products. *Subsidiary or secondary production* of an industry is defined by exclusion as *non*-principal production.

4. As can be seen from Gigantes' equation 7, (our equation 2.81).

5. One should bear in mind here that Gigantes' original formulation of 2.64 was a prelude to the development of a theory of *joint* products, which are not considered in this paper. Separate demand conditions are distinguished for such products in Gigantes' paper, and bi-products are considered to be uninfluenced by demand at all. (Principal products are the dog and bi-products the tail; the possibility of the latter wagging the former is presumably ruled out by fixed relative prices hypothesised in the model.)

6. The use of set theory notation here precludes the more general case in which M, etc., may be divided so that only *part* of the ijth element is assigned a particular technology assumption. This notation is used merely for simplicity of exposition and the results derived apply equally validly to the more general case.

7. Results obtained in equations 2.74, 2.76 and 2.77 are identical with those in Gigantes' paper. I presume his proofs are therefore roughly the same, though to my knowledge they have not been published.

8. Although it is possible that the r_{kj} in the relation $a_{ij} = \sum_k b_{ik} r_{kj}$ vary in offsetting ways with the b_{ik} and the a_{ij} constant, there seems to be no *a priori* theoretical justification for such an assumption.

CHAPTER 2:

R E F E R E N C E S

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CHAPTER 3:

ECOLOGIC-ECONOMIC INPUT-OUTPUT

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3-1 INTRODUCTION

In this chapter we describe the adaptation of the Leontief, Stone and Activity Analysis systems to the study of economic-environmental interactions in the national economy. Leontief's original conception ([1], [2]) was of the use of an augmented I - O system to project pollution magnitudes consequent upon different patterns of final demand for economic commodities, and to evaluate each pollutant in terms of its per unit abatement costs. We shall show, however, that the model possesses much wider potential for analysis in this field, drawing on and criticising the work of other writers in the process.

Activity Analysis is also shown to be a technique amenable to adaptation for ecologic analysis and provides some method (albeit a not altogether satisfactory one) of evaluating the short-run opportunity costs of ecologic commodities in the absence of data on abatement costs.

3-2 THE LEONTIEF MODEL

Leontief's model of the relation between the economic system and the ecologic system in which it is embedded is a reasonably simple development from equation 2.7 above. Let the original economic input-output coefficients matrix of that equation be represented by

A_{11}. Leontief proceeds to border this matrix by three other matrices A_{12}, A_{22}, and A_{21} to form an augmented coefficients matrix of different, but equal, dimensions. These matrices are defined as follows:

$A_{12} = [a_{ig}]$ shows the input of economic commodity i per unit of abated pollutant g (abated by sector g);

$i = 1, \ldots m;$

$g = m + 1, \ldots n$

$A_{21} = [a_{gi}]$ shows the output of pollutant g per unit of output of economic commodity i (produced by sector i);

$i = 1, \ldots m;$

$g = m + 1, \ldots n$

$A_{22} = [a_{gk}]$ shows the output of pollutant g per unit of abated pollutant k (abated by sector k);

$k, g = m + 1, \ldots n$

3-2.1 Production and Pollution

Balance equations for the augmented system may be written in matrix form as follows:

$$\left[\begin{array}{c|c} I - A_{11} & -A_{12} \\ \hline A_{21} & A_{22} - I \end{array}\right] \left[\begin{array}{c} q_1 \\ \hline q_2 \end{array}\right] = \left[\begin{array}{c} f_1 \\ \hline f_2 \end{array}\right] \tag{3.1}$$

where

q_1, q_2 and f_1, f_2 represent respectively vectors of total output and final demand for economic commodities and pollution abatement.

$I - A_{11}$ is thus the familiar economic net coefficients matrix of equation 2.17. However, since pollution abatement is not costless in terms of economic commodities required for its operation, it is necessary to incorporate into the balance equation the matrix A_{12} showing the per unit absorption by abatement sectors of economic commodities. The first 'row' of 3.1 is therefore

$$(I - A_{11})q_1 - A_{12}q_2 = f_1 \tag{3.2}$$

which shows that total output of any given economic commodity must now be sufficient to cover not only intermediate and final usage of economic sectors but also the intermediate consumption of abatement sectors. The second 'row' of 3.1 may be written

$$A_{21}q_1 + A_{22}q_2 - q_2 = f_2 \tag{3.3}$$

affirming that pollution from production activity *plus* pollution from abatement activity *minus* the quantity abated must equal the amount 'supplied' to final demand, i.e. 'tolerated' by households. For comparison with 2.17 equation 3.1 may be written thus:

$$q^* = A^*q^* + f^* = (I - A^*)^{-1}f^* \tag{3.4}$$

where

$$A^* = \left[\begin{array}{c|c} A_{11} & A_{12} \\ \hline -A_{21} & 2I - A_{22} \end{array}\right]$$

and

$$f^* = \begin{bmatrix} f_1 \\ -- \\ f_2 \end{bmatrix} \qquad\qquad q^* = \begin{bmatrix} q_1 \\ -- \\ q_2 \end{bmatrix}$$

(the coefficient of I being the scalar 2).

As 3.4 makes clear, the conditions for the inverse of $(I - A^*)$ to exist are not identical with those for the existence of $(I - A)^{-1}$ given in equations 2.15 and 2.16 above —— for example, A^*, unlike A, contains negative elements.

Existence of the inverse $(I - A^*)^{-1}$ being a necessary condition for a unique solution to the above equation, A^* must be a square matrix. Thus we cannot allow more pollutants than abatement sectors.

3-2.2 Pricing and Pollution

The Leontief price system incorporating abatement activities cannot be expressed as the simple *dual* problem to 3.4 above, analogously to the price and quantity systems of equations 2.18 and 2.19. Writing the augmented price system in the form

$$p^* = Ap^* + y^* = (I - \tilde{A})^{-1} y^* \qquad (3.5)$$

where

$$\tilde{A} = \left[\begin{array}{c|c} A'_{11} & Q'_{21} \\ \hline A'_{12} & Q'_{22} \end{array}\right]$$

$$Q'_{21} = \left[q_{gi}\right] = \left[r_{gi}a_{gi}\right]$$

$$i = 1, \ldots m$$

$$g,k = m + 1, \ldots n$$

$$Q'_{22} = \left[q_{gk}\right] = \left[r_{gk}a_{gk}\right]$$

$$y^* = \begin{bmatrix} y_1 \\ -- \\ y_2 \end{bmatrix} \qquad p^* = \begin{bmatrix} p_1 \\ -- \\ p_2 \end{bmatrix}$$

y_1, y_2 being vectors of value-added created respectively in production and abatement activity;

p_1, p_2 being vectors of shadow prices of respectively economic and ecologic commodities; and

Q'_{21}, Q'_{22} being scaled down versions of the original pollution coefficients matrices transposed (viz. A'_{21} and A'_{22}), the proportions being r_{gi} and r_{gk},

$i = 1, \ldots m;$

$g,k = m + 1, \ldots n;$

then the first 'row' of 3.5 may be expanded as

$$p_1 - A'_{11}p_1 - Q'_{21}p_2 = y_1 \tag{3.6}$$

—— which is to say the price of an economic commodity must be just sufficient to cover the cost of intermediate economic inputs, ecologic abatement and primary inputs.

The second 'row' of 3.5 is likewise expressed as

$$p_2 - A'_{12}p_1 - Q'_{22}p_2 = y_2 \tag{3.7}$$

asserting that the price of an *ecologic* commodity should be adequate to cover the costs of economic commodities and primary inputs used per unit of ecologic commodity abated, *plus* the 'ecologic costs' per unit of abatement (i.e. the imputed economic costs of ecologic commodities generated per unit of each ecologic commodity eliminated). To clarify this point we write down a typical element of equation 3.7:

$$p_k^2 - \sum_i a_{ik} p_i^1 - \sum_g q_{gk} p_g^2 = y_k^2 \qquad (3.8)$$

in which numerical superscripts indicate the appropriate sections of the price vector p^*, and primary input vector y^*. From these simultaneous relationships it is clear that shadow prices derived for economic and ecologic commodities will depend on the proportion of unit emissions from production (equation 3.6) and abatement (equation 3.7) activities that it is decided to eliminate, and indeed that such proportions are independent of the levels of pollution abatement specified in the vector of final demand. From which it follows that the total amount of pollution abated, whilst clearly affecting the total pollution bill, has no influence on the pricing of individual pollutants, i.e. on their relative 'economic impact'.

The maximum values Q_{21} and Q_{22} may have are A_{21} and A_{22} respectively, when the reduction factors are set equal to unity; their minima are reached when the factors assume the value zero. This latter situation implies that the anti-pollution sectors are redundant, for their outputs are necessarily zero; and zero activity levels cannot generate demand for economic commodity and primary inputs. In this event equation 3.7 vanishes and we are left with 3.6 in reduced form:

$$p_1 - A'_{11}p_1 = y_1 \tag{3.9}$$

which is equivalent to 2.19 with $A'_{11} = A$, $p_1 = p$, and $y_1 = y$. We also have $f_2 = 0$ and $A_{21}q_1 + A_{22}q_2 = q_2$.

3-2.3 Recycling

Recycling of materials from production and abatement activity, and from consumption, are important aspects of a general policy of conservation. Matter cannot be created or destroyed, and therefore residuals from abatement activity are merely transformed into (hopefully) more manageable forms, e.g. from airborne to waterborne types. However, such material may also be recovered for use in the industry generating the residuals or for sale to other industries. This process is described as one of recycling of materials for the result is the production of economic commodities (even though further residuals are inevitably generated in the process of materials-energy conversion).

Leontief's ecologic-economic system can incorporate the phenomenon of recycling in principle as we shall show. But before doing so it is necessary to refute some arguments which suggest the contrary.

Victor ([3] pp.49-50) distinguishes two forms of recycling: (a) as "done by a particular industry after purchasing the necessary equipment and technique from the anti-pollution industry", and (b) "by means of a waste merchant who sorts and processes waste so that it can be re-used as industrial input". Both these forms of recycling are, he argues, precluded by Leontief's assumptions: (a), because the technical coefficients must change; and (b), because "if recycling is actually done by the anti-pollution industry, the industry must not only be paid to collect the waste, but it must sell the waste, after any necessary treatment, back to industry". But these objections are absurd. Firstly, there is no question of recycling requiring the input-output coefficients of the model to 'change' (we construe the term change here as meaning change over time since no other connotation makes sense in the context). Secondly, the latter objection seems to be a claim that recycling in the Leontief system requires multi-product industries and that this is inconsistent with Leontief's hypothesis that industries and products are one-to-one. We shall show, however, that the existence of multiple-product industries does not present *practical* problems, even if the interpretation of the system leaves something to be desired.

Let r_{ij}^{11} and r_{ik}^{12} be the quantity of the ith commodity recycled per unit of the jth industry and kth abatement sectors' outputs respectively. Then the equation for the typical element of the 'revised' version of equation 3.2 is:

$$q_i^1 - \sum_j (a_{ij}^{11} - r_{ij}^{11})q_j^1 - \sum_k (a_{ik}^{12} - r_{ik}^{12})q_k^2 = f_i^1 \qquad (3.10)$$

If the input-output coefficients of this model are defined as $a_{ij}^* = a_{ij} - r_{ij}$, then there is evidently no need to make any assumption about the constancy or otherwise of the a_{ij} or r_{ij}, though empirically their behaviour should be similar. So far the implicit assumption has been that the r_{ij}'s have been measured in *value* terms at base-period prices, for in the context of 3.10 the choice of mass units would make the equations impossible to interpret because of incommensurabilities. Now, if we have $r_{ij} > a_{ij}$, any $i \neq j$, this means that the industry in question is a multi-product industry, because it is now, due to the phenomenon of recycling, producing subsidiary commodities in conjunction with its principal product. This eventuality makes it seem more rational to regard a_{ij} and r_{ij}, rather than merely a^*_{ij}, (all i,j), as (constant) parameters of the system.

Recycling may, therefore, destroy the one-to-one correspondence of commodities and industries, but unless it involves defining new commodities, no fresh variables are introduced into the system thereby, and so a unique solution is still possible. Of course the problem of allocating each industry's purchases amongst the now multifarious sources of supply of any given commodity remains; but, as mentioned, this is a problem of interpretation, not specifically of operationality, since however the question is decided it cannot affect the solution of the equations, i.e. alter the total outputs of commodities and abatement sectors required to satisfy a specified bill of final demand.

In matrix notation the model may be written in the same form as 3.4 above with

$$A^* = \left[\begin{array}{c|c} A_{11} - R_{11} & A_{12} - R_{12} \\ \hline -A_{21} & 2I - A_{22} \end{array}\right] \tag{3.11}$$

and corresponding price equations as in 3.5 and

$$\tilde{A} = \left[\begin{array}{c|c} A'_{11} - R'_{11} & Q'_{21} \\ \hline A'_{12} - R'_{12} & Q'_{22} \end{array}\right] \qquad (3.12)$$

In these equations the matrices R_{11} and R_{12} are general, i.e. not diagonal, matrices.

We may thus conclude that the Leontief system can deal operationally with the question of recycling of materials from productive uses. A simple extension of the above reasoning demonstrates the possibility of incorporating materials recycled from consumptive uses in the model, the matrices R then being defined to include materials from consumption sectors. We shall see later that in some respects the more flexible assumptions of the Stone accounting framework permit a more 'definite' theoretical interpretation of the process, and that to this extent the model is superior to the present one.

3-2.4 Trade-off Between Ecologic Commodities

Leontief points out that the elements of the A_{22} matrix, showing the quantity of a given ecologic commodity generated per unit of some ecologic commodity abated, exhibit the trade-off between the levels of output of different ecologic commodities. This is a logical consequence of the proportionality assumption of the Leontief system: the relative magnitudes of pullutants generated per unit of output and abated per unit of output are not a function of the activity level of the sector in question. From this it should not be inferred that not any vector of final tolerances can *theoretically* be achieved with a given coefficients matrix A*. For, although the amounts of other pollutants i (i = m + 1, . . . n) generated by sector j per unit of j's output are fixed, these ecologic commodities may be abated by other sectors $i \neq j$. Of course this argument assumes, very reasonably, that $1 - a^{22}_{jj} > 0$, j = m + 1, ... n; i.e. that the abating sector does in fact eliminate more of the pollutant it abates than the quantity of that pollutant it generates in the process. Because this is true for *any* sector j, the proposition follows. The sense in which a trade-off exists is that in the process of eliminating one pollutant we inevitably generate others, which, for a given vector of final tolerances of all pollutants, implies that *more* than simply the industrially generated quantities of each ecologic commodity must be removed to satisfy these final tolerance limits. What the present version of the Leontief model does not do however, is to draw attention to the existence of a trade-off between ecologic commodities in another sense: namely, between the mass of materials distributed between different ecologic sinks (such as Land, Air and Water). Trade-offs of this kind arise as a consequence of the Principle of the Conservation of Matter, and the fact that recycling is never 'perfect', - in the sense of the Second Law of Thermodynamics. These important notions have been discussed in detail in chapter 1 above. There is no theoretical reason why a pollutant should not be the subject of an indefinitely long series of abatement processes, but due to the fact that each such process itself involves matter-energy transformations it is not ecologically costless. Furthermore, in the Leontief model it is assumed, as we have noted, that any final tolerance vector can be achieved: there are no resource limitations. Since in practice limitations may exist (e.g. labour or capital may be in short supply if the economy is running at capacity) an *economic* trade-off is also implicit.

3-2.5 Shadow Prices and Unit Taxes

Pollution may be conceived as an externality, as a cost imposed on society by industrial production (and consumption) of economic commodities, and not charged in the final prices of products. Ecologic externalities of this kind may be 'internalised' by charging the costs of pollution abatement to producers via pollution unit taxes. Looking back at equations 3.6 and 3.7 we can see that the appropriate levels for these unit taxes should be equal to the ecologic shadow prices of the augmented Leontief system, since they reflect the economic costs in terms of intermediate and primary commodities required for each unit of an ecologic commodity abated. Abatement activity may be either privately or publicly run; in the former case, the government may make a law that the costs of abating a certain proportion of each industry's pollution must be charged by abatement sectors to the industry in question. In the latter situation, the government exacts the taxes and channels the money into the department concerned with eliminating pollution.

This method of "making the polluters pay" entails that the pollution bill for an industry for a given pollutant is strictly proportional to the amount of its ecologic discharge: if an industry produces twice as much SO_2 as before then its tax bill doubles; if it reduces its discharges by half, its tax bill halves; and so on. However, unless the Leontief system is divested of its economic-theoretical character and simply reduced to a statistical hypothesis regarding the behaviour of certain economic aggregates, it is not possible to view unit pollution taxes as 'optimal' in the marginalist sense of this term. In this model marginal costs are constant with respect to variations in output, as are marginal revenues; firms cannot therefore be conceived as altering output levels to bring the two into equality, since if marginal revenue is not already equal to marginal cost there is no mechanism within the system available to achieve this condition. Putting the matter another way, firms in the Leontief economy produce commodities, economic and ecologic, independently of their marginal costs. Prices of products used and produced by industry are simultaneously determined by the set of input-output coefficients and industry value-added. Output levels are determined by the same set of input-output coefficients in conjunction with a fixed pattern of final demand. But prices of inputs and outputs are independent of sectorial activity levels.

3-2.6 Effects on Level and Distribution of Income

Total income in any period, Y, may be written

$$Y = y'q = (w + \pi)'q \tag{3.13}$$

with w and π respectively vectors of industry wages and profits coefficients; the change in national income from one period to another, denoted ΔY, we can use Leontief's analytical framework to calculate this latter as

$$\Delta Y = Y_p - Y_o = (w_p + \pi_p)'q_p - (w_o + \pi_o)'q_o \qquad (3.14)$$

subscripts o and p indicating base-period and projection-period quantities respectively. The shares of wages and profits (S_w and S_π) in the national income may then be written

$$S_w = \frac{w'q}{(w + \pi)'q} \quad \text{and} \quad S_\pi = 1 - S_w \qquad (3.15)$$

with absolute changes in these quantities (ΔS_w and ΔS_π) as

$$\Delta S_w = S_{w(p)} - S_{w(o)o} \quad \text{and} \quad \Delta S_\pi = -\Delta S_w \qquad (3.16)$$

and proportional changes

$$\frac{\Delta S_w}{S_w} = \frac{S_{w(p)} - S_{w(o)}}{S_w} \quad ; \quad \frac{\Delta S_\pi}{S_\pi} = \frac{-\Delta S_w}{1 - S_w} \qquad (3.17)$$

The effect of pollution abatement policy on the level and distribution of income can now easily be written down. We have, using the notation of equation 3.5:

$$\Delta Y = (y_{1p}, y_{2p})' \begin{bmatrix} q_{1p} \\ --- \\ q_{2p} \end{bmatrix} - (y_{10}, y_{20})' \begin{bmatrix} q_{10} \\ --- \\ q_{20} \end{bmatrix} \qquad (3.18)$$

and if no pollution is abated in the base-period (i.e. $y_{20} = q_{20} = 0$) and other economic activity is unaltered (i.e. $y'_{1p}q_{1p} = y'_{10}q_{10}$)

$$\Delta Y = y'_{2p}q_{2p}$$

The change in the share of wages due to pollution abatement is then

$$\Delta S_w = \frac{w'_{1p}q_{1p} + w'_{2p}q_{2p}}{(w_{1p},w_{2p})'q_p + (\pi_{1p},\pi_{2p})'q_p} - $$

$$- \frac{w'_{10}q_{10}}{(w_{10},w_{20})'q_o + (\pi_{10},\pi_{20})'q_o} \tag{3.19}$$

from which the change in the share of profits and the respective proportionate changes in both shares can be obtained via equations 3.16 and 3.17.

The rationale of calculating the effects on the distribution of income of a policy of 'ecologic husbandry' should be quite obvious to anyone familiar with current debates over the morality of relative distributive shares, and it is equally clear that the future importance of such issues is not likely to decline.

3-2.7 Ecologic Discharges from Final Demand

Pollution does not end at the factory gate in more ways than one: the process of industrial production of economic commodities is not the only source of ecologic residuals because of the inexorable operation of the Law of Entropy whose functioning entails that no commodity can have 'eternal life' [5]. In other words, so-called final consumption of economic commodities tends to generate waste both as bi-product during use and as final residuals at the end of the commodity's life. Thus, for example, a motor car wears out various of its components and consumes petrol while serving its owner as a means of transportation, and at the end of its life must be bodily disposed of. Rarely is all the material of which the car is made recycled, and therefore quantities of metal, plastic, rubber, glass etc., are discharged to the environment. As regards *airborne* residuals from the final use of commodities, Leontief's system accommodates them in the following manner.

Let $a_{g,y(i)}$ be the output of ecologic commodity g generated by the final use of commodity i delivered to consumption.

This defines a matrix $A_y = [a_{g,y(i)}]$ of pollution coefficients applying to each commodity supplied to final demand.

Let $\tilde{f}_2$ be a vector of ecologic magnitudes g (= m + 1, . . . n) generated by final usage of economic commodities.

Then we must have

$$\tilde{f}_2 = A_y f_1 \tag{3.21}$$

with f_1 as in 3.1 above. In other words the total amount of the gth ecologic commodity generated by final usage is the sum of the quantities produced by the final use of each economic commodity by households. Leontief in effect, therefore, proposes rewriting equation 3.3 in the following fashion:

$$A_{21}q_1 + A_{22}q_2 + \tilde{f}_2 - q_2 = f_2 \tag{3.22}$$

That is, industrially-generated pollution, *plus* consumption-generated pollution, *minus* the quantity abated, equals the quantity eventually supplied to, or tolerated by, the household sector. To solve this system define a new vector f_2^* such that

$$f_2^* = f_2 - \tilde{f}_2 \tag{3.23}$$

and substitute this in equation 3.1 above.

We noted in section 3-2.2 that shadow prices in the Leontief system are determined by the unit absorption coefficients of industries including primary inputs, and not by the total amounts of economic commodities consumed or ecologic commodities abated. Prices are also unaffected by the total outputs of economic and ecologic commodities. Hence relative prices of commodities are unaffected if the total quantity of pollution is augmented by quantities arising directly from household consumption. Of course the total pollution bill for a given level of environmental quality (vector of final tolerances) will be larger the larger the total amount of each ecologic commodity produced since this implies (as is manifest from equation 3.22) that abatement sectors must be operating at higher levels. Symbolically, for non-null $\tilde{f}_2$ and given tolerance vector f_2, for any element $\tilde{f}_{2i}$ of $\tilde{f}_2$ greater than zero we must have the corresponding element of q_2, i.e. q_{2i}, $> q^*_{2i}$, its value for $\tilde{f}_{2i} = 0$.

3-2.8 Employment Repercussions of Conservation Policy

Let labour input coefficients for the set of environmental sectors be denoted by m_e such that

$$m_e = [\overbrace{0,0, \ldots 0}^{m \text{ elements}}; \quad m_{m+1}, \ldots m_n] \tag{3.24}$$

then total employment L generated directly and indirectly by conservation policy is given by

$$L = (\hat{m}_e q)'1 \tag{3.25}$$

with $\hat{m}_e = \text{diag}(m_e)$ and q the vector of total outputs in the projection period.

The relevance of these calculations to environmental policy is justified by a similar argument to that adduced in 3-2.6.

3-2.9 Total Ecologic Magnitudes

Gross pollutant outputs from economic activity, i.e. production and consumption, may be calculated from the components of the Leontief system as follows.

Let b represent a vector of gross pollutant outputs from production and consumption activity. This vector may be decomposed, according to equation 3.22, thus:

$$b = \left[A_{21} \vdots A_{22} \right] q^* + f_2 - q_2 \qquad (3.26)$$

with q^* as in equation 3.4.

This vector b shows the volume of each ecologic commodity that would be associated with $q_2 \geq 0$ in equation 3.22; in other words, with **abatement $\geq$ 0.** Even in the absence, therefore, of data on pollution abatement costs the Leontief model may be utilised in conjunction with a matrices of ecologic coefficients from production and consumption activity, A_{21} and A_y, together with a projected pattern of final demands for economic commodities, to determine the ecologic magnitudes associated with these demands.

3-2.10 Ecologic Impact Tables

Ecologic magnitudes associated with a particular economic commodity are a function not merely of the amount of economic commodity supplied directly to final demand but also of the quantities supplied to other industries for intermediate usage. To consider the relative industrial ecologic impact of individual commodities it is necessary to normalise for differences in levels of final supply, since total impact as given in equation 3.26 above is clearly a weighted sum which includes the f_i as partial weights. Furthermore it is of interest to examine the impact of *each* economic commodity on the production of each pollutant. Thus, instead of writing, say

$$s = A_{21}(I - A_{11})^{-1}1 \tag{3.27}$$

with the on-line 1 representing the summation vector, showing the direct and indirect effect of supplying one unit of each economic commodity to households on the magnitude of a given pollutant, we write, with Leontief,

$$S = A_{21}(I - A_{11})^{-1} \tag{3.28}$$

$$= A_{21} + A_{21}A_{11} + A_{21}A_{11}^{2} + \ldots\ldots + A_{21}A_{11}^{n-1} + \ldots$$

the typical element of which matrix S, s_{ij}, showing the quantity of the ith ecologic commodity directly and indirectly associated with the industrial supply of the one unit of the jth economic commodity to final demand.

Whilst Leontief's impact matrix is a correct index of the relative *industrial* impact of each economic commodity, it quite neglects the further ecologic consequences of consuming these units, on the environment. Equation 3.28 should thus be rewritten as

$$S = A_{21}(I - A_{11})^{-1} + A_y \tag{3.29}$$

In some contexts it may be of more interest to know not the numerical impact of each economic commodity as given by the elements of S but rather their *proportional* contributions. This can be expressed by premultiplying S by the inverse of diag(s*), the vector of equation 3.27, to which has been added A'_y, diagonalised:

$$S^* = \hat{s}^{*-1}S \tag{3.30}$$

A typical element of S^*, s^*_{ij}, shows the proportion of the total amount of the ith pollutant generated by the supply of one unit of each economic commodity to final demand, that is contributed by the jth sector.

3-3 MEADE'S ANALYSIS

J.E. Meade in a recent article [6] suggests that insofar as the Leontief model is to be employed in conjunction with some kind of consumer demand function it seems preferable to conceive of the economy as supplying to final consumption not a vector of pollution tolerances but a certain (scalar) quantity of 'Clean Air'. The argument implicitly assumes that the demand function in question relates the wants of consumers directly to the commodities in question, as opposed to mediating those wants through some representative body (e.g. a democratically elected government) which may have or employ the expertise sufficient to relate those wants to the Leontief vector of pollution tolerances. Meade rightly emphasises that consumers themselves are concerned not so much with the specific quantities of substances removed from the air, but rather with the quantity of the end-product of abatement activity that is available.

In Meade's system, therefore, we write, analogously to Leontief's equation (3.3),

$$-\sum_{1}^{n} a_{n+1,j}q_j - a_{n+1,n+1}q_{n+1} + q_{n+1} + \bar{q}_{n+1} = f_{n+1} \qquad \ldots\ldots \quad (3.31)$$

where

$a_{n+1,j}$	is the input of clean air (the (n + 1)th commodity) per unit of output of industry j;
$a_{n+1,n+1}$	is the input of clean air per unit of output of the Clean Air sector, i.e. the sector producing clean air;
q_j	is the output of the jth industrial sector (j = 1, . . . n);
f_{n+1}	is the quantity of clean air supplied to final demand; and
$\bar{q}_{n+1}, q_{n+1}$	are respectively the initial stock of clean air and the quantity of clean air 'produced' by the sector of that name.

Although it is nowhere stated by Meade how the quantity "clean air" is to be measured, it is presumably intended that units such as square or cubic miles of air*space* conforming to certain ecologic criteria of purity (e.g. having a vector of ecologic concentrations attached to each unit) are to be employed.

One of the essential differences between the approaches of Meade and Leontief is that for Leontief the objective of the system is to evaluate economic commodities and the *flow* of pollution, whereas for Meade the task is to evaluate economic commodities and the *stocks* of clean air and human leisure, together with the flow of clean air as the output of economic activity.

Meade's pricing circuit may be written

$$(I - A')p = p^* \tag{3.32}$$

in which

- A is an $(n + 1)(n + 1)$ input-output coefficients matrix including the clean air sector both as user and producer of clean air;
- p is an $(n + 1) \times 1$ vector of prices of economic commodities *and* the price of clean air (the $(n + 1)$th price);
- p^* is the product of a $(n + 1) \times 1$ vector of labour coefficients, m, measured in man-hours or some suitable equivalent, and the 'price' of labour or the wage-rate, p_{n+2}, measured in terms of a numéraire commodity. It is thus a vector of primary input coefficients.

The procedure Meade follows in relating price and quantity circuits (see further on for a description of this latter) is to assume an identical utility function for all members of the community:

$$U = U(f, f_{n+2}) \tag{3.33}$$

(the arguments respectively being a vector of final demands for commodities including clean air, and final demand for leisure). This objective function is then maximised subject to the community budget constraint:

$$B = \bar{q}_{n+1}p_{n+1} + \bar{q}_{n+2}p_{n+2} - f'p - f_{n+2}p_{n+2} = 0 \quad \ldots\ldots \tag{3.34}$$

that is, the sum of the values of the stocks of clean air and man-hours equals the value of total expenditure on economic commodities and clean air $(f'p)$, and leisure $(f_{n+2}p_{n+2})$.

This procedure results in $2(n + 1) + 1 = 2n + 3$ equations (these are $n + 1$ equations from the maximisation problem) in $2(n + 2) = 2n + 4$ unknowns. However, the value of the wage-rate is, *ex hypothesi*, arbitrary, so that the system will in normal circumstances yield a unique solution for the prices of economic commodities and the price of clean air, p, and the price of leisure, p_{n+2}, together with the vectors of final demands for these quantities.

Meade's quantity circuit may be written

$$q - Aq = f - \bar{q} \tag{3.35}$$

or

$$(I - A)q = f - \bar{q} \tag{3.36}$$

and

$$m'q = \bar{q}_{n+2} - f_{n+2} \tag{3.37}$$

where q is an $(n + 1)$ - vector of commodity outputs including clean air, and

$$\bar{q} = (\overbrace{0,0\ .\ .\ .\ 0}^{n\ \text{elements}},\ \bar{q}_{n+1}).$$

With the solution values of the maximum problem substituted in the vector f this system constitutes a set of $n + 2$ equations in $n + 1$ unknowns. It is, however, as Meade shows, easy to see that only $n + 1$ of these equations are independent (in fact we have $p'(I - A)q = p^{*\prime}q = p'(f - \bar{q})$).

3-3.1 Supply, Demand and Price

An interesting and important feature of Meade's analysis is that the prices of commodities - economic and ecologic - are determined by supply and demand. In the Leontief system, as we have seen, quantities, and in particular quantities of commodities supplied to final demand, are independent of prices. Meade's analysis, however, treats the vectors p, f, and f_{n+2} as independent variables, and their values determined simultaneously as the outcome of a consumer's utility maximisation problem subject to constraints. Prices of economic and ecologic commodities are thus implicitly assumed to reflect marginal social rates of substitution between products.

3-3.2 Clean Air as a Public Good

Meade now proceeds to show that if we regard clean air as a public good, government anti-pollution policy leads to exactly the same result as the operations of the market we have just described. Assume the government solves 3.32 for p_{n+1}, the price of clean air; call this $\tilde{p}_{n+1}$; and solves 3.36 for q_{n+1}, the corresponding quantity, call this $\tilde{q}_{n+1}$. The government leaves the production of clean air to private enterprise but purchases $\tilde{q}_{n+1}$ units for its citizens, revenue required to accomplish this being raised by levying a vector of unit taxes, $t = (t_1, \ldots t_n, 0)$ on industrial sectors (i.e. excluding the Clean Air activity). Total revenue from this source, R, is thus

$$R = t'q \tag{3.38}$$

Taxes are set at rates $t_j = a_{n+1,j}\tilde{p}_{n+1}$, so that t is the product of the (n + 1)st row of the A matrix (the vector of clean air input coefficients) with a zero in the (n + 1)st place, and the price of clean air, $\tilde{p}_{n+1}$.

i.e.

$$R = (a_{n+1}q)\tilde{p}_{n+1} \tag{3.39}$$

where

$$a_{n+1} = [a_{n+1,j}], \text{ a row vector.}$$

Revenue raised from this source will thus be proportional to the quantity of clean air consumed by industrial sectors. Any surplus of revenue over expenditure is distributed by the government in the form of lump-sum taxes to the citizens.

Price equations may now be written:

$$p = A'p + p^* + t - a'_{n+1}\tilde{p}_{n+1} \tag{3.40}$$

or

$$(I - A')p = mp_{n+2} + t - a'_{n+1}\tilde{p}_{n+1} \tag{3.41}$$

which constitutes a system of (n + 1) equations in (n + 1) - 1 + 1 = (n + 1) unknowns, viz. the p_i, $i = 1, \ldots n$; and p_{n+2}. With $U = U(f, f_{n+2})$ citizens take f_{n+1} as given (= $\tilde{f}_{n+1}$) and maximise utility subject to the new budget constraint

$$B = p_{n+2}\bar{q}_{n+2} + (t'q - p_{n+1}\tilde{q}_{n+1}) - (p'f - \tilde{p}_{n+1}f_{n+1}) = 0$$

$$\ldots\ldots \quad (3.42)$$

Meade shows that the resulting set of equations yields precisely the same price set as the system 3.32 (the mathematics is unfortunately not easily translated into matrix notation and for that reason we omit it here).

Finally, Meade discusses the situation where the government allows producers to pollute the atmosphere and then taxes citizens to raise revenue to 'clean up' the pollution. He concludes that this solution to the problem must be generally suboptimal, since "[i]n this case cost-prices have been distorted from their "ideal" levels because pollution costs are not ascribed to the various activities. Consumers' demands and so outputs will now be affected both by the constraint that they must consume a given amount of clean air [determined by the volume of pollution generated] and also by the fact that the relative prices of . . . marketable [economic] goods will no longer correspond to their social costs" ([6] p.152).

3-3.3 Criticism of Meade's Analysis

It may be as well to begin by allaying a possible misconception as to the interpretative validity of Meade's approach. For whilst it is manifest that each sector in the model *requires* in a technological sense, clean air to produce its output of economic commodities, it might seem that it is only via the institution of government law that an industrial sector is obliged to *purchase* clean air, including the output of the Clean Air sector. This, however, is erroneous. The (positive) price for clean air is determined completely by technological conditions and consumer demand functions, as equations

3.32 through 3.34 show. Thus Meade is perfectly correct in describing this as a "market" solution if we give the elements of the input-output system their usual interpretation.

We turn now to more relevant criticisms of Meade's approach.

(i) A difficulty arises with the idea of consuming clean air, i.e. buying the output of the Clean Air sector or depreciating the initial stock of this ecologic commodity, because it does not follow that a sector has reduced the stock of clean air merely from the fact that it has discharged a (non-null) vector of ecologic commodities in the period under consideration; the environment's assimilative capacity acts as a (set of) variable(s) determining the relation between the output of ecologic commodities and the quantity of clean air available. Because of the form of this relation it seems comparatively less plausible to assume the per unit absorption of clean air is fixed over the period considered than to make this assumption about the per unit discharge of ecologic commodities.

(ii) Secondly, though Meade's conception of clean air is as a stock *plus* a flow, he does not, curiously enough, regard the total quantity of clean air, and hence the total quantity of air (clean plus polluted), as fixed. His equations show the total amount of clean air 'available for final enjoyment by citizens or use by industry' as the sum of 'the initial stock of clean air' *and* 'the amount produced by the anti-pollution activity'. The level of operation of the Clean Air sector, like all Leontief-type sectors, is not, *per se*, subject to resource limitations. Meade's stock of clean air is not, in consequence, a constraint parameter in the solution of the system but merely a constant addition to the quantity of the ecologic commodity produced by the Clean Air sector: the higher the value of the 'initial stock' the less this sector has to produce to satisfy a given bill of final demand for this commodity. For this reason the stock of clean air serves a disparate function from the stock of man-hours available to society, for this latter is a true primary commodity, not produced within the system at all, and whose total is absolutely fixed: here the only choice available to society is whether to consume this fixed quantity as work or leisure; once the amount of work or leisure is decided upon the amount of leisure or work is simultaneously determined. Now, although at low levels of clean air consumption it may be perfectly reasonable to regard the quantity of the ecologic commodity as unconstrained, it is surely evident that, if we consider the units of measurement of the good to be say cubic feet of airspace a certain level above ground, the total amount of this resource available to the citizens of the world, and of any particular economy, is definitely fixed. Thus, on the one hand, citizens cannot demand more than a certain amount of this environmental good; and on the other, anti-pollution activity is physically incapable of producing more than a limited quantity of it. (This argument assumes, of course, that the commodity in question is kept at a constant quality throughout.) Whilst it may be argued that for 'normal' levels of consumption and output there is unlikely to be any effective limitation on the quantity of clean air produced and consumed, there is no guarantee that in 'the short run' this will necessarily be true (unless 'the short run' is *defined* as the period for which it is true, - in which case it *may* be too short to be useful).

The issue this criticism raises is largely one that may be decided only by empirical testing of the model. In fact it seems quite likely that the model would be operational in this respect, strictures of the Club of Rome [4] against the finitude of all resources not withstanding.

(iii) Meade's formulation of the community utility function, though adopted merely for reasons of simplicity of exposition ([6] p.147, note), does not emphasis sufficiently the interdependence of utilities, a factor first emphasized by Duesenberry [10]. This is not, however, a serious criticism of the system since the point could be relatively easily accommodated in an operational sense, either (a) if the functional forms of the individual welfare indicators were known, or (b) if the social welfare function were decided by a paternalistic authority who settled the issue by means of relative individual weightings in that social welfare indicator.

(iv) Meade is lax in failing to distinguish 'final consumption' of clean air in the sense of (a) final enjoyment which does *not* pollute the air, thus leaving the quantity of clean air available to households unchanged after use; and (b) final enjoyment which reduces (or, conceivably, increases) the quantity available to households. This latter phenomenon can, however, be easily accommodated by addition of a term representing the quantity of clean air used up by final demand to the left-hand side of equation 3.31.

3-4 THE STONE SYSTEM

3-4.1 Commodity and Industry Ecology

Reasoning along similar lines to those employed in our discussion of commodity and industry technology, it seems appropriate to question, in the context of a model which distinguishes commodity and industry outputs, whether ecologic coefficients should be considered as dependent upon industries or commodities. Light on this issue is thrown by invoking the symbolism of section 2-4.5. The coefficient denoting the output of the ith ecologic commodity per unit of output of the kth economic commodity when the latter is produced in the jth industry has three subscripts. The addition of this third dimension is a consequence of the many-one assumption concerning the relation of commodities and industries in the Stone System, for, unlike the Leontief model, Stone's accounting framework permits the possibility of the output of pollution from the production of one unit of a given commodity varying according to the industry in which the commodity is manufactured. Thus, steel manufactured by the Steel industry may produce less pollution per unit of output than steel produced by, say, the Iron industry due to the fact that in the former it is produced in such quantities that more efficient controls are required by the Alkali Inspectorate on its emissions, and that the volume of experience in controlling such emissions is much greater in the industry of which steel is the principal product.

By analogy, therefore, with technology concepts introduced in the Stone model, we may distinguish two broad ecological structure assumptions:

Commodity ecology (C.E.) is the hypothesis that the ecologic structure of commodities is independent of industries, and can be expressed formally by the set of equations

$$\pi_{ik} = \beta_{ik1} = \beta_{ik2} = \ldots = \beta_{ikn} \quad \text{any } i,k \qquad \ldots.. \quad (3.43)$$

in which β_{ikj} is the output of the ith ecologic commodity from the kth economic commodity, the latter produced in the jth industry.

Industry ecology (I.E.) is the hypothesis that the ecologic structure of commodities is determined by their industries of origin and may be written

$$\pi_{ij} = \beta_{i1j} = \beta_{i2j} = \ldots = \beta_{inj} \quad \text{any } i,j \qquad \ldots.. \quad (3.44)$$

Hence, under the dispensation of a commodity ecology we have a pollution x commodity coefficients matrix $\Pi = [\pi_{ik}]$; and under that of an industry ecology there results a pollution x industry coefficients matrix $\Pi = [\pi_{ij}]$. As was shown to be possible with commodity input structure assumptions with respect to economic commodities, one can envisage a medium between these two hypothetical extremes by allowing that the pollution structure of some economic commodities may be determined by the producing industry and others independently of industries. We term such an assumption the *hybrid ecology* hypothesis, (H.E.) and, just as in the economic analysis we divided the Make matrix up into two additive components for the purpose of assigning commodity outputs to different input technology assumptions, so here we envisage a subdivision of the Make matrix in accordance with respective output ecology hypotheses for various commodities.

Just as output structural assumptions are not completely independent of input technology, so also are ecology assumptions related to input structure. For, if we assume that the amount of pollution produced per unit of output of a commodity is a function of the quantity of various commodities *absorbed per unit* of its output (a perfectly reasonable hypothesis from a physical point of view, which is the language in which technology is discussed), then evidently if the input structure of commodities is independent of industries we should expect the ecologic structure to be independent; and conversely, if the input structure of commodities is determined by industries we should expect the ecologic structure to be so determined. Thus in an empirical sense we have, roughly:

$$\text{C.T.} \longleftrightarrow \text{C.E.} \qquad \text{and} \qquad \text{I.T.} \longleftrightarrow \text{I.E.}$$

In consequence, it seems inconsistent to employ the C.T. assumption unless we assume that pollution from a given industry is a weighted sum of the outputs of commodities it produces, the weights being the ecologic coefficients. Symbolically,

$$p_{ij} = \Sigma \pi_{ik} m_{jk} \tag{3.45}$$

where p_{ij} denotes the output of the ith ecologic commodity from the jth industry. I.T. similarly implies the proportionality of pollution to industry outputs:

$$p_{ij} = \pi_{ij} g_j \tag{3.46}$$

It also follows from this line of reasoning that ideally a division of the Make matrix into component parts for technology purposes should, in the context of an environmental model, reflect the choice of ecology assumptions for those outputs.

3-4.2 Ecologic Impact Matrices

Consider an I.T.-I.E. model with matrices of ecologic coefficients *re* production and consumption, $\Pi^{\text{Ind.}}$ and $\Pi^{\text{Cons.}}$ defined by

$$\Pi^{\text{Ind.}} = P\hat{g}^{-1} \tag{3.47}$$

and

$$\Pi^{\text{Cons.}} = W\hat{f}^{-1} \tag{3.48}$$

where

$g = [g_j]$ is, as before, a vector of *industry* outputs;

$f = [f_i]$ is a vector of *commodity* consumption levels;

$P = [p_{kj}]$ is a matrix of ecologic magnitudes from industry outputs;

$W = [w_{ki}]$ is a matrix of ecologic magnitudes from final consumption of economic *commodities*.

Notice that $\Pi^{\text{Ind.}}$ is of dimensions ecologic commodity x industry, whilst $\Pi^{\text{Cons.}}$ is of dimensions ecologic commodity x economic commodity. In order to combine the ecologic impacts of production and consumption activity a transformation matrix is therefore required. Our interest centering primarily on the ecologic impact of commodities, this transformation matrix will be of dimension industry x economic commodity and will perform the function of adding in the quantities of pollution 'indirectly' generated by the supply of one unit of each economic commodity to final demand. We shall call this matrix U . The resulting impact table is then expressed by the equation

$$S = \Pi^{Ind.}U + \Pi^{Cons.} \tag{3.49}$$

U is easily defined in terms of known economic matrices as

$$U = (I - E_I)^{-1}T \tag{3.50}$$

3-4.3 Ecologic Magnitudes

A vector of total ecologic outputs b_η is got from equation 3.49 by simply recalling the definition of an impact matrix S and consequently postmultiplying by the vector of final demand for commodities, f :

$$\begin{aligned} b_\eta &= Sf \\ &= S_I f + S_c f \qquad (3.51) \\ &= P1 + W1 \end{aligned}$$

with

$$S_I = \Pi^{Ind.}U \qquad \text{and} \qquad S_c = \Pi^{Cons.}$$

Once again this derivation presupposes the validity of the I.T.-I.E. hypothesis. An analogous set of equations may be inferred on the C.I.-C.E. assumption; in this situation the transformation matrix serves merely to add in the indirectly generated pollution, not to convert from industries to commodities.

In a hybrid ecology context, as mentioned, the outputs of commodities in the Make matrix must be segmented into two appropriate groups which should be identical with a division made according to technology assumptions. Thus if the set of commodity outputs to be treated on a C.E. are given by M_1 and those to be treated on an I.E. by M_2, we can write

$$M_1'1 = q_1 \tag{3.52}$$

$$M_2 1 = g_2 \tag{3.53}$$

then industrial pollution from C.E. and I.E. outputs is expressed by the following equations:

$$\delta_1 = \Pi_1 q_1 \tag{3.54}$$

$$\delta_2 = \Pi_2 g_2 \tag{3.55}$$

with

$$\Pi_1 = [\pi^1_{ik}]$$

$$\Pi_2 = [\pi^2_{ij}]$$

matrices of pollution coefficients *re* commodities and industries respectively, and

$$\delta_1 = [\delta^1_i]$$

$$\delta_2 = [\delta^2_i]$$

vectors of ecologic magnitudes from C.E. and I.E. outputs. The vector of total industrial pollution magnitudes δ_H, is obviously the sum of δ_1 and δ_2 :

$$\delta_H = \delta_1 + \delta_2 \tag{3.56}$$

It should be noted here that because C.T./I.T. outputs do not constitute different categories of final demand some relation between q_1 and g_1 and q and g must be assumed in order to calculate ecologic outputs under hybrid ecology.

Pollution impact and ecologic magnitudes from final consumption activity are calculated in the same way as before.

3-4.4 Shadow Pricing of Economic Commodities

The bifurcation into commodity and industry outputs introduced by the Stone System implies in an I.T. system that we evaluate the economic impact of pollution abatement policy on industries rather than commodities (as in the C.T. system) since here the prices of commodities produced in a given industry, being identical, are each affected in precisely the same way.

3-5 TIME AND THE ECOLOGIC-ECONOMIC PROCESS

'Internalisation' of pollution by the imposing of pollution charges or legal limits on emissions levels (or consequent ambient-air densities) presents the entrepreneur with a reformulated optimization problem: the production function is initially the same, but now subject to an augmented set of constraints. Short-run objectives, then, must be subject to fixed technological parameters, - the form of the production-function; the longer-term permits consideration of these parameters as decision-variables, operable upon by changing techniques. Thus, from the ecological perspective, the entrepreneur's long-range objective is to (say) minimize costs subject to (say) absolute levels of discharge of SO_2 and Particulates, and so he will select from the spectrum of production techniques available to him that piece of equipment which conforms most satisfactorily with the totality of his criteria, economic and ecologic. Now, since ex hypothesi, discharge limits exist on only two of the (several) ecologic commodities his enterprise generates, he is free to adopt a technique whose pollution coefficients re SO_2 and Particulates are low relative to the remainder. Expressing the matter in more familiar jargon, the business man may substitute one pollutant for another (hopefully, less toxic, but certainly less prohibited and/or costly).

3-5.1 Time-Dependence of I - O Coefficients

It is widely known that the economic coefficients of the Leontief and Stone models are time-dependent over long periods. The same is true with possibly more force of ecologic processes. It is not to be expected for both technical and ecologic-economic reasons, then, that an input-output system will yield accurate predictions over a year or more unless this aspect of structure is taken into account.

One of the tasks to which an I - O system incorporating environmental linkages may be directed is the analysis of changes in national ecologic magnitudes over several years. In particular, this can be done in retrospect, to determine the effectiveness or otherwise of government policy to abate pollution. For example, we might consider the effectiveness of the 1956 Clean Air Act, and the precise nature of the causes bringing about changes in ecologic magnitudes experienced. There are basically four kinds of determinants of changes in ecologic magnitudes that may be considered; changes in:

i the pattern of final demand;
ii the level of final demand;
iii the economic I - O structure;
iv the ecologic I - O structure.

The method by which these are analysed is that of seeing what changes in pollution levels would have occurred over any given time period if one of these quantities had alone varied whilst the other three had been constant. For example, if we are determining the effect of ecologic structural change between t and $t + n$, then the matrix of such changes, ΔR , is defined as

$$\Delta R = \bar{R}^{t+n} - R^{t} \tag{3.58}$$

with

$$\bar{R}^{t+n} = \Pi_I^{t+n}[(I - E)^{-1}Df]^{t} + S_c^{t+n}\hat{f}^{t}$$

$$R^{t} = S^{t}f^{t}$$

superscripts referring to the time-period for which the matrices are calculated.

3-5.2 Substitution Amongst Pollutants

It is obviously also possible to analyse, via changes in technical and ecologic coefficients, the extent of substitution over time. In the case of ecologic coefficients we should expect certain kinds of pollution policy to favour the substitution of one fuel for another - say oil for coal - and this will generally affect not merely the relative outputs of say SO_2 and Particulates but also that of the other pollutants associated with the combustion of the different kinds of fuel. By this means, then, the extent of a trade-off in any given sink can be calculated. If data is available on discharge coefficients to other sinks it is possible to achieve a considerably more accurate assessment of environmental policy, because certain techniques of abatement reduce air pollution at the expense of increasing water pollution (e.g. particulate collector devices using wet washers). Mathematical formulae for the extent of substitution over a period can be defined as follows.

Take any arbitrary pollutant as 'standard' and express the ecologic output coefficients of the jth sector as proportions of this standard pollutant for any two periods t and t-n ;

for period t :

$$\frac{\pi^t_{1j}}{\pi^t_{1j}}\ , \quad \frac{\pi^t_{1j}}{\pi^t_{2j}}\ , \quad \ldots\ldots \quad \frac{\pi^t_{1j}}{\pi^t_{pj}} \tag{3.59}$$

or

$$S^t_{11}\ , \quad S^t_{12}\ , \quad \ldots\ldots \quad S^t_{1p} \tag{3.60}$$

with

$$S^t_{kh} = \pi^t_{kj}/\pi^t_{hj}$$

(the j subscript being omitted in S^t_{kh}). For period t-n we have a similar set (S^{t-n}_{kh}) . The degree of substitution between pollutants k and h over the period t-n to t is simply

$$\Delta S_{kh} = S^t_{kh} - S^{t-n}_{kh} \tag{3.61}$$

or in proportional form:

$$\Delta S^*_{kh} = \Delta S_{kh}/S^{t-n}_{kh} \quad \text{(say)} \tag{3.62}$$

Obviously, since ecologic commodity 1 is standard we do not need to calculate ΔS for it. With regard to the case $k \neq h$, we have the following:

$$0 < \Delta S^*_{kh} \quad \text{means } k \text{ substituted for } h \text{;}$$

$$\Delta S^*_{kh} = 0 \quad \text{means no substitution;}$$

$$\Delta S^*_{kh} < 0 \quad \text{means } h \text{ substituted for } k \text{.}$$

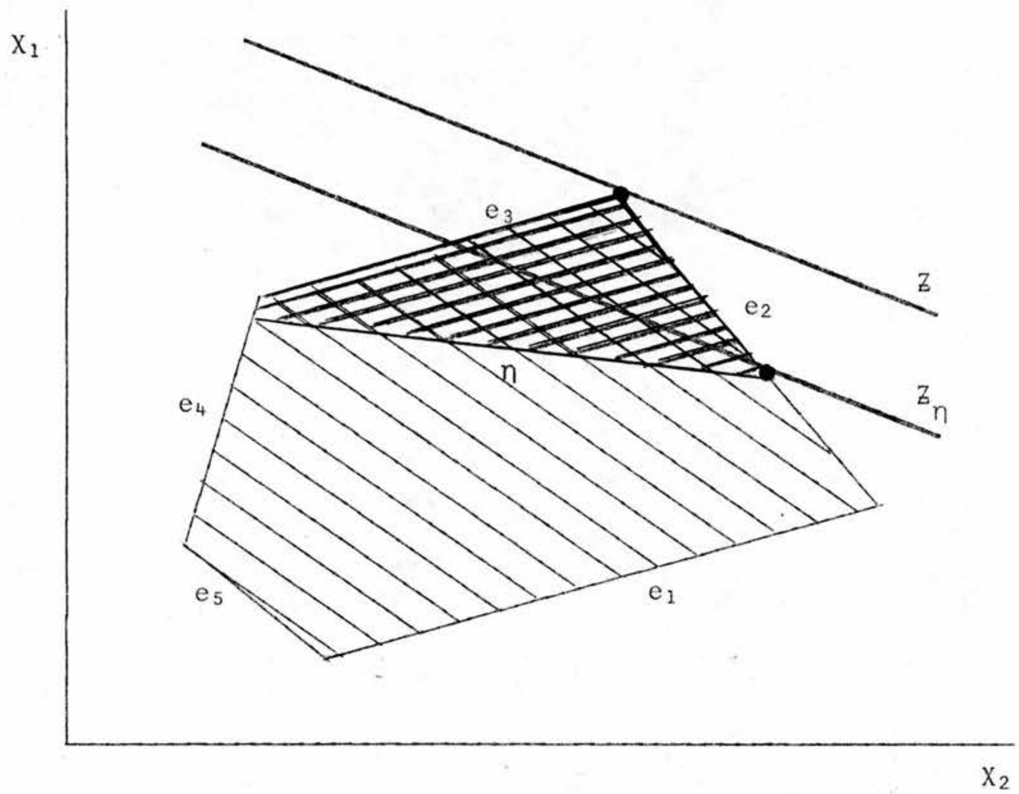
Likewise, if we wish to know the rate of substitution between any other two pollutants, say ℓ and h ($\ell \neq k$), this is obtained from the ratio $S_{k\ell}/S_{kh}$.

3-6 ACTIVITY ANALYSIS MODELS

3-6.1 Introduction

The apparatus of Activity Analysis described in section 2-5, like the Leontief and Stone systems, is readily amenable to adaptation for the study of economic-ecologic linkages. This is initially achieved by the introduction of a further set of constraints on the levels that may be obtained by the activities determining the value of the objective function. For each ecologic commodity generated by the economic activities of production and consumption we specify an upper limit, or a discharge constraint. Any economic optimum - maximum of consumption or minimum of cost - must accommodate this set of ecologic constraints, so that the (extreme) point of the optimum can only lie within a convex *subset* of the convex set defined by the economic region. The complement of this subset is the <u>ecologically prohibited region</u>. The simple two-dimensional case is elucidated in Figure 7 below.

<u>Figure 7</u>



We have five economic constraints (labelled e , . . . e_5) and one ecologic constraint (η) . The optimum value of the objective function in the absence of ecologic constraint is labelled Z , and subject to ecologic limitations, Z_η . The light shaded area is the economically feasible region; the dark shaded area the ecologically prohibited region.

Models involving ecologic constraints were developed for the purpose of regional analysis by Isard [7], and Russell and Spofford [8] and Russell [9]. Analysis of national ecologic-economic interactions commenced with the Rosenbluth models applied by Victor [3] to the Canadian economy in 1972. Only this last is of direct relevance to the present study and we shall engage in a brief critical discussion of its analytical characteristics.

The Rosenbluth model takes as its objective the minimization of combined ecologic and economic costs of the national economy subject to minimum prespecified levels of final demand. Whilst the economic weights of the criterion function were readily obtainable, no such data was forthcoming on ecologic unit costs. Recourse was therefore had to an ad hoc estimation procedure which we shall see has rather unsatisfying theoretical credentials.

Ideally, in any national policy decision, we should know society's indifference surface as a function of both economic and ecologic commodities. If society's indifference surface including ecologic commodities is known, then its marginal rate of substitution between every pair of commodities and pollutants can be calculated and optimum levels of consumption and abatement determined. In Victor's analysis a set of ecologic unit weights are devised to represent the marginal social valuation of (any) one unit of each of several ecologic commodities. He employed a chemist to decide the relative magnitude of these weights taking into account the pollutants' toxicity, ability to disperse, and interact, their colour, odour, etc.

In general the social valuation of a commodity (economic, ecologic) will depend on the quantities of other commodities consumed. For example, the principle of diminishing marginal rate of substitution implies that the more of one economic commodity consumed the less that has to be relinquished to obtain a marginal increment of another economic commodity if the consumer (and, by extension, society) is to remain indifferent. In the case of ecologic commodities whose effect is to produce <u>dis</u>utility, the same kind of logic may be considered to apply; both viz à viz other ecologic commodities and with respect to economic commodities (though marginal rates of substitution will have opposite signs in the two categories). One very obvious reason for interdependence amongst valuations of ecologic commodities is provided by the existence of synergistic effects amongst pollutants in the environment, since these imply that the toxicity of any given substance depends on the ambient concentration of other substances. The upshot of these considerations is that any theory purporting to evaluate pollutants from the point of view of their tendency to generate receptor damage cannot rationally represent the social welfare function as additively separable in its arguments. Furthermore, due to the existence of threshold effects with rising marginal disutility, even if separability is allowed, it seems highly likely that non-linearities will be involved. In general, if the welfare function is additively separable and takes the simple form

$$W = \Sigma W^j(X_j) = \Sigma w^j(X_j)X_j \tag{3.63}$$

where W^j and w^j are functions only of X_j, the quantity of economic/ecologic commodity j consumed, then it can also be written as

$$W = \Sigma\left[X_j\frac{dW^j}{dX_j} - X_j^2\frac{dw^j}{dX_j}\right] \tag{3.64}$$

with the special case w^j = constant ^{j} as

$$W = \Sigma X_j \frac{dW_j}{dX_j} = \Sigma X_j w^j \tag{3.64a}$$

In general, however, the terms $X_j^2 \frac{dw^j}{dX_j}$ are clearly nonzero so that welfare will be higher $(\frac{dw^j}{dX_j} < 0)$ or lower $(\frac{dw^j}{dX_j} > 0)$ than in 3.65. Unfortunately it is precisely the form 3.65 that Victor has adopted to represent the welfare damage-effects of ecologic commodities in his model.

Though perhaps substantial in respect of the interpretative validity of his own applications of the Rosenbluth models, these objections against Victor's weighting system do not vitiate the use of (specifically) linear programming techniques for environmental purposes. No substantial theoretical objections exist against the employment of a separable objective function, optimised subject to a set of constraints. Indeed, such models have already been successfully developed and applied by Russell and Spofford and others in America. Again, it is worth emphasising that the essential nonlinearities and interdependencies of MSV's of ecologic commodities do not impugn the perfectly valid use of constant per unit abatement costs in determining shadow prices of ecologic commodities; only the interpretation placed on these quantities is affected: it is not possible to consider them as accurately reflecting the cost imposed on society by the phenomenon of pollution. In general it seems probable that total ecologic cost will be underestimated by this method, though this is a matter for speculation at present.

To Victor's ecologic weights as means of evaluating the social cost of pollution there are alternatives. In section 3-6.2 we describe two models which evaluate different ecologic commodities by means of shadow prices reflecting the short-run impact of a 'crash'-type pollution policy. In the very short run the only way of meeting a set of ecologic requirements is by curtailing output. Ecologic shadow prices then reflect the marginal effects of relaxing such standards in an optimally arranged economy. Over a longer period these 'ecologic indices' will be inaccurate arbiters of value as the possibility of changing ecologic coefficients reflecting the instalment of abatement equipment, less polluting processes, etc., becomes realised.

The models of section 3-6.3 explicitly allow for the possibility of short-run abatement programs to satisfy pollution standards by introducing in like manner to the Leontief System, an anti-pollution sector, assumed over the short run to be capable of meeting any ecologic demands made upon it. Such an assumption can, of course, be relaxed by the introduction of further constraints. In the case of some of the original ecologic constraints being effective at higher levels of abatement than is physically possible from the anti-pollution sector shadow prices will once again (partially) reflect requisite cutbacks in industrial output.

3-6.2 Models Without an Abatement Sector

The fundamental ecologic assumption underpinning the environmental activity analysis models is the proportionality of ecologic magnitudes to inputs/outputs from industries, and to consumption levels in final demand sectors. Symbolically, if $P = [p_{ij}]$ represents a matrix of ecologic magnitudes re production activity, then

$$A_{\eta} = P\hat{g}^{-1} \tag{3.65}$$

where:

$A_{\eta} = [a^{\eta}_{ij}]$ = a matrix of ecologic input-output coefficients re production activity levels $(a^{\eta}_{ij} \leqslant 0)$.

Respecting consumption sectors, the ecologic hypothesis is rendered as:

$$E_{\eta} = W^{*}\hat{k}^{-1} \tag{3.66}$$

where

$E_\eta = [e^\eta_{ij}]$ = a matrix of ecologic coefficients re consumption sectors $(e^\eta_{ij} \leqslant 0)$;

$W^* = [w^*_{ij}]$ = a matrix of ecologic magnitudes generated by consumption activity;

$k = [k_j]$ = a vector of consumption sector activity levels.

By rearranging equations 3.65 and 3.66 we can derive an expression for base-period ecologic magnitudes from economic (production and consumption) activity as the product of a partitioned matrix of ecologic emission factors and a column vector of economic activity levels, similarly partitioned:

$$\left[P \;\middle|\; W^* \right] 1 = \left[A_\eta \;\middle|\; E_\eta \right] x = \tilde{b}_\eta \qquad (3.67)$$

with $x = [g \, , \, k]$ and $\tilde{b}_\eta$ a vector of base-period ecologic magnitudes from economic activity.

The imposition of discharge limits on ecologic commodities generated by the economic system may now be represented by a set of inequalities:

$$\tilde{b}_\eta \geqslant b_\eta \qquad (b_\eta, \, \tilde{b}_\eta \leqslant 0) \qquad (3.68)$$

$-b_\eta$ is a vector of upper bounds on the discharge of ecologic commodities from production and consumption. Ecologic matrices 3.65 and 3.66 are incorporated in the economic activity analysis models 1 and 2 by augmenting the input-output coefficients set for each activity. Typical production and consumption activities now take the form:

$$a_j = \begin{bmatrix} a^E_j \\ \hline a^\eta_j \\ \hline m_j \end{bmatrix} \qquad e_j = \begin{bmatrix} e^E_j \\ \hline e^\eta_j \\ \hline 0 \end{bmatrix} \qquad (3.69)$$

Hence models 2.124 and 2.125 above lend themselves to rewriting in the following augmented fashion:

(i) Maximize $I = (0', 0', 1')x$

S.T.

$$\left[\begin{array}{c|c|c} A_E & E_E & c \\ \hline A_\eta & E_\eta & c_\eta \\ \hline m' & 0' & 0' \end{array}\right] x \geqslant \left[\begin{array}{c} 0 \\ \hline b_\eta \\ \hline b_L \end{array}\right] \tag{3.70}$$

$$x \geqslant 0$$

(ii) Minimize $S = (s', 0')x$

S.T.

$$\left[\begin{array}{c|c} A_E & E_E \\ \hline A_\eta & E_\eta \\ \hline m' & 0' \end{array}\right] x \geqslant \left[\begin{array}{c} b_E \\ \hline b_\eta \\ \hline b_L \end{array}\right] \tag{3.71}$$

$$x \geqslant 0$$

The first 'rows' of the constraint set in models 3.70 and 3.71 are identical with those of 2.124 and 2.125 respectively. The second 'rows' of 3.70 and 3.71 may be set out more explicitly as:

$$A_\eta g + E_\eta k + c_\eta k_f \geqslant b_\eta \tag{3.72}$$

and

$$A_\eta g + E_\eta k \geqslant b_\eta \tag{3.73}$$

which (bearing in mind the definition of b_η) assert that each ecologic magnitude from production and consumption activity combined, should not exceed prespecified levels. Introduction of these sets of ecologic constraints facilitates the evaluation of ecologic commodities on a short-run basis, in the absence of data on value-added created by the activity of pollution abatement and a set of input-output coefficients re economic commodities related to such a sector. To see this an examination of the interpretation of the duals to 3.70 and 3.71 is required.

The duals to programs 3.70 and 3.71 are:

(iD) Minimize $R = (0', b_\eta', b_L)p$

S.T.

$$\left[\begin{array}{c|c|c} A_E' & A_\eta' & m \\ \hline E_E' & E_\eta' & 0 \\ \hline c' & c_\eta' & 0 \end{array}\right] p \leqslant \left[\begin{array}{c} 0 \\ \hline 0 \\ \hline 1 \end{array}\right] \tag{3.74}$$

$$p \geqslant 0$$

and

(iiD) Maximize $V = (b_E', b_\eta', b_L)p$

S.T.

$$\left[\begin{array}{c|c|c} A_E' & A_\eta' & m \\ \hline E_E' & E_\eta' & 0 \end{array}\right] p \leqslant \left[\begin{array}{c} s \\ \hline 0 \end{array}\right] \tag{3.75}$$

$$p \geqslant 0$$

In (iD) we seek to minimize national resource cost, now conceived as the weighted sum of the limits on economic and ecologic 'primary' supplies. In (iiD) the desideratum is that of maximising the net surplus of the value of commodities supplied to final demand, over their resource cost, - now defined to include ecologic components.

3-6.3 Models with an Abatement Sector

As mentioned, in these models the brunt of abatement policy is not felt solely by industries in terms of lost output. Here industries may purchase quantities of 'negative pollution' so that in aggregate the control policy constraints are satisfied. Since technically the addition of an abatement sector to models i and ii is virtually the same in both cases we shall describe only the analysis for the first model.

The activity vector for the abatement sector is denoted by α :

$$\alpha = \begin{bmatrix} \alpha_E \\ \hdashline \alpha_\eta \\ \hdashline m_\alpha \end{bmatrix} \tag{3.76}$$

where α_E is a vector of economic input coefficients;

α_η is a vector of net ecologic coefficients (i.e. showing the amount of pollution eliminated per unit of output net of the amount produced);

m_α is a primary input coefficient for the abatement sector.

Inserting this vector into model i of the previous section yields:

(iii) Maximize $I = (0', 0', 0', 1)x$

S.T.

$$\left[\begin{array}{c|c|c|c} A_E & \alpha_E & E_E & c \\ \hline A_\eta & \alpha_\eta & E_\eta & c_\eta \\ \hline m & m_\alpha & 0' & 0' \end{array}\right] x \geqslant \left[\begin{array}{c} 0 \\ \hline b_\eta \\ \hline b_L \end{array}\right] \tag{3.77}$$

$$x \geqslant 0$$

The second 'row' of the constraint set is thus

$$A_\eta g + \alpha_\eta r + E_\eta k + ck_f \geqslant b_\eta \tag{3.78}$$

Remembering the adopted sign conventions, we shall require that α_η, r (the activity level) $\geqslant 0$, and the equation states that pollution from production and consumption net of abatement should not exceed some prespecified standard.

The dual program is now

(iiiD) Minimize $R = (0', b_\eta, b_L)$

S.T.

$$\left[\begin{array}{c|c|c} A'_E & A'_\eta & m \\ \hline \alpha'_E & \alpha'_\eta & m_\alpha \\ \hline E'_E & E'_\eta & 0 \\ \hline c' & c'_\eta & 0 \end{array}\right] p \leqslant \left[\begin{array}{c} 0 \\ \hline 0 \\ \hline 0 \\ \hline 1 \end{array}\right] \tag{3.79}$$

$$p \geqslant 0$$

The addition of an abatement sector thus results in a new equation to be satisfied by the vector of shadow prices, viz.,

$$\left[\alpha_E', \alpha_\eta', m_\alpha \right] p \leq 0 \tag{3.80}$$

which is exactly analogous to the price equations for economic sectors.

3-7 SUMMARY

In this Chapter we have seen how Leontief has adapted the traditional static open input-output system to the study of economic-environmental interactions, how his analysis can be extended to determine the effects of anti-pollution policy on the level and distribution of national income and the volume of employment. We have shown that the Leontief ecologic impact table which hitherto had neglected the consumption impact of commodities is easily amended to incorporate this feature. Meade's input-output analysis with Clean Air as a public good was also expounded and criticised. The contribution of Stone's system to environmental input-output analysis via the notions of commodity and industry ecology which we have developed by analogy with his technology concepts was demonstrated. The influence of temporal changes in the various structural parameters of the environmental input-output model was examined, showing that by using certain techniques the independent effects of changes in technology, ecology, patterns and levels of final demand and substitution amongst pollutants could be isolated, thus facilitating important policy evaluations.

Finally, the Activity Analysis of the environmental implications of economic processes was demonstrated to have useful properties. Even in the absence of a viable social welfare function (analytically conceived) or abatement cost data Activity Analysis could be employed to evaluate the impact of a crash programme of pollution control.

CHAPTER 3:

R E F E R E N C E S

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CHAPTER 4:

AN APPLICATION OF THE ENVIRONMENTAL INPUT-OUTPUT MODEL

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APPLICATION OF THE ENVIRONMENTAL INPUT-OUTPUT MODEL

4-1 PAST AND PRESENT STUDIES

In chapter three we described in some detail the adoption of various linear national economic models to the study of environmental problems. The prime objective of this chapter will be to demonstrate by means of example the mode of application and usefulness of two of these models in the U.K. context. The application is best termed an empirical exercise in the methodology of pollution control at national level; for reasons which will become apparent as the study proceeds, the results obtained depend to a considerable degree on guided conjecture. At each stage some attempt is made to check and correct the data employed, and an exhaustive search for information to support adopted hypotheses is implemented. This notwithstanding huge gaps in our knowledge do at present exist and our estimates will accordingly suffer from a larger margin of error than is generally admissible. Justification for such a venture can only reasonably be sought in the stimulus it may provide to other interested parties to improve and develop the basic information and raw materials of the ecologist's trade. We will see the worthwhileness of the kind of achievement of Leontief in America with this quality of data. It is to be hoped, then, that other workers will take up the lead laid down in that and the present study.

4-1.1 Previous Empirical Work in Environmental Input-Output: A Summary of the Literature

Only three studies have used environmental input-output models on a nationwide scale, those of Victor [10], Leontief and Ford [11] and Mathur [13]. They will be discussed in that order.

Victor applied a Leontief-Stone-type model and a Linear Programming model to Canada for the year 1961. Flows of four types of water input and twenty-seven types of water, air and land output for sixteen economic sectors and final demand were estimated. Labouring under considerable difficulties in obtaining base-period ecologic data, especially for water inputs and outputs, Victor was forced into making some rather heroic assumptions. The data was applied to estimate the ecologic impact of each economic commodity (i.e. one unit's direct and indirect generation of a vector of pollutants) and, by employing a putative set of social weights (for a discussion of which see section 3-6.1), to determine the relative ecologic cost of these economic commodities. Using the same weights Victor applied a national Linear Programming model to the Canadian Economy having as objective the minimisation of total ecologic costs involved in the supply of a specified vector of final demand for goods. Finally, Victor attempted to estimate, using both types of model, the economic and ecologic implications of a 50% switch from private to public transportation.

We shall not here enter into a discussion of Victor's estimates of ecologic cost as their theoretical validity has been impugned in an earlier section (3-6.1)

The main defect in Victor's empirical work lies in the paucity of data: predictions are never any better than the assumptions on which they are based, and many of Victor's estimates of industrial process emissions rely on information which is not sufficiently process-specific, uses outdated coefficients estimates, or which lacks details of control efficiencies applicable at the time. Greater relative informational reliability was obtained for the air and land sinks; data for water inputs stands out as the most extravagantly conjectural, being based on a far-fetched analogy with American industrial sector inflows ([10], p.103) since no actual data for Canada in 1961 was available.

These criticisms notwithstanding Victor's study constituted the first serious and tenacious attempt to quantify the empirical and Social impact of economic activity on a national scale via the ecosystem feedback link. Our own application of similar models which follows later in the chapter can only in certain respects be considered more accurate in its assumptions. However, it should also be pointed out that in some respects at least it is less ambitious in its aims.

Leontief and Ford [11] applied Leontief's model to the American economy over the years 1958, 1963, 1967, and projected trends forward to 1980. The economic data of a governmental growth project was utilised to determine the ecologic impact of 90 sectors, 30 of which were shown in detail, the remainder in broader aggregates. Five air pollutants were considered, no data apparently being available on other waste flows, and the coefficients, derived on 1967 data only, were assumed invariant over the relevant periods. Leontief and Ford perform the following set of things:

1. Calculate an ecologic impact table (based on 1967 data). Call this matrix S .

2. Calculate the air pollution content of eleven final demand vectors. In matrix notation, if $f_1, \ldots f_{11}$ are the final demand vectors, then the direct and indirect pollution contents per unit of each sector's demand are given by

$$Sf_1/f_1'1, \quad Sf_2/f_2'1, \; \ldots \ldots \; Sf_{11}/f_{11}'1$$

3. Project (on an 83-industry classification) ecologic magnitudes to 1980 levels, assuming constancy of 1967 ecologic coefficients. This is analysed further by:
   (a) assuming constant technical (economic) coefficients, and aggregate final demand, allowing for variations in demand patterns;
   (b) assuming constant (1958) final demand patterns and levels and allowing for changing (economic) technology.

4. Estimate of the price effects of an anti-pollution program based on data for wage and interest costs of pollution control processes (no data was available on other - intermediate - inputs). Price effects were calculated for four hypothetical control strategies for 90 industries. The strategies considered included the substitution of low- for high-sulphur fuels.

Leontief's results for objectives 3 and 4 are very interesting. For example, over the 22-year period Leontief's figures show that changing final demand composition alone (with fixed technological coefficients) would have decreased the amount of airborne particulates generated by the Food sector by 268 thousand tons, whilst increasing the quantity generated by the Utilities sector (our "Nationalised Industries") by 244 thousand tons. Similarly, over the same period Leontief's data show that changing technology alone (with a fixed bill of goods) would have increased particulates from the Food sector by a mere four thousand tons, whilst from the Utilities sector the expected increase is 1259 thousand tons. This implies that for the Food sector with regard to particulate pollution changing economic technology is some 67 times more important a source of changing pollution magnitudes than changing final demand patterns whilst in Utilities changing demand is merely 19 per cent as important as changing technology. This kind of comparison highlights the potentiality of input-output in the study of pollution problems since the central government can by fiscal means influence the composition of final demand or technological change, and it is important that the relative costs and environmental impact of so doing should be able to be evaluated rationally.

Mathur [13] in an exploratory study of the use of Leontief input-output model to trace the repercussions of pollution control considers the economic effects of a U.K. air pollution policy of 'making the polluter pay', i.e. charging the cost of control to private firms. Utilising the available U.K. data on (a) costs already incurred by industries to this purpose, and (b) their expenditure plans in the same area, Mathur calculates for 1968, in an analogous way to Leontief, the price effects of the additional factor (primary) inputs to these industries and other industries consuming their products. For this purpose, since data was available only on aggregates of CSO sectors, the breakdown was into a 65 x 65 industry/commodity classification. In addition by treating imports as factors of production, i.e. primary inputs, Mathur was able to determine the pollution cost avoided by importation (this is based on the notion that there is pollution only from the production of imports, and this is done abroad. However, the consumption-ecologic effects should strictly be considered as part of the 'bargain'). Secondly, Mathur calculates the effects of additional working costs required (in the estimation of the Alkali Inspectorate) to bring the control of selected industries up to a specified standard (again given by the Inspectorate), on the cost of 1968 output, private consumption, government consumption, fixed capital formation, exports and import replacements. Finally, an attempt is made to compare the relative progress of the U.K. and America viz à viz bearing the cost of pollution control measures. Taking the American rate of expenditure on pollution control measures for 1967 (designed to achieve 1975-specified pollution standards) Mathur determines the effects that would occur if a similar programme were to be adopted here.

Regarding empirical results, Mathur found that the 1968 cost of national output was increased by a minimum of 0.08% due to historically implemented control measures, rising to 0.18% with the implementation of anticipated further expenditures to attain Inspectorate-specified standards, including a partial desulphurization of petroleum. Considering individual sector impacts, it was found that the private consumer bore less than the average, at the rate of 0.5% and 0.15% respectively, with public consumption and fixed capital bearing a slightly higher burden of control costs. Examining the impact of control on exports Mathur concluded that though this class of commodities bore a slightly higher-than-average cost no conclusions with regard to the effect on international competitiveness could be drawn in the absence of data on pollution control policy of other countries.

He was able to deduce, however, that on average exports and imports of the U.K. are not sensitive to pollution control measures by calculating the hypothetical effect of producing (i.e. replacing) all U.K. imports with the same pollution control as given by costs already incurred to that purpose; the result was indicated by the fact that the increase in export costs was less than the increase in import costs from replacement and hypothetical uniform control. In all this international comparison Mathur is at pains to emphasize that we are only concerned with the differential effect on U.K. industries; any equi-proportionate impact may be obviated by lowering the exchange rate accordingly. In general conclusion he calls for more work to be done on the empirical side by economists if results on pollution abatement are to be assessed more accurately.

4-1.2. Aims of the Present Study

We now outline five main objectives to be achieved:

1. To use the secondary ecologic coefficients for industries to derive (i) (unit) ecologic impacts, and (ii) ecologic magnitudes, for economic commodities, from production.

2. To calculate the consumption impact of economic commodities and their related magnitudes.

3. To outline some general characteristics of the relation between direct and indirect pollution produced by the supply of economic commodities, and suggest possible policy implications arising therefrom.

4. To rank each economic commodity by (i) production, and (ii) consumption (unit) impact and total magnitudes for certain major pollutants.

5. To examine the correlation over all economic commodities between unit and total impact for each of several major pollutants. (As explained later, this will show the (average) relative importance of final demand (total) requirements in producing observed national pollution magnitudes, in the absence of time-series data.)

We now present, for the reader's convenience, a listing with correspondent numbering, of the economic and ecologic commodities used in this study.

Table 4.1

Numbering of Economic Sectors/Commodities in CSO 1968 Input-Output Tables

	Industry/commodity group
1	Agriculture
2	Forestry and fishing
3	Coal mining
4	Stone, slate, chalk, sand, etc. extraction
5	Other mining and quarrying
6	Grain milling
7	Other cereal foodstuffs
8	Sugar
9	Cocoa, chocolate and sugar confectionery
10	Oils and fats
11	Other food
12	Soft drinks
13	Alcoholic drink
14	Tobacco
15	Coke ovens and manufactured fuel
16	Mineral oil refining, lubricating oils and greases
17	General chemicals
18	Pharmaceutical chemicals and preparations
19	Toilet preparations
20	Paint
21	Soap and detergents
22	Synthetic resins, plastic materials and synthetic rubber
23	Dyestuffs and pigments
24	Fertilizers
25	Other chemical industries
26	Iron castings, etc.
27	Other iron and steel
28	Aluminium and aluminium alloys
29	Other non-ferrous metals
30	Agricultural machinery
31	Machine tools
32	Pumps, valves and compressors
33	Industrial engines
34	Textile machinery
35	Construction and mechanical handling equipment
36	Office machinery
37	Other non-electrical machinery
38	Industrial plant and steel work
39	Other mechanical engineering
40	Instrument engineering

(cont.)

Table 4.1 (cont.)

Numbering of Economic Sectors/Commodities
in CSO 1968 Input-Output Tables

Industry/commodity group	
41	Electrical machinery
42	Insulated wires and cables
43	Electronics and telecommunications
44	Domestic electrical appliances
45	Other electrical goods
46	Shipbuilding and marine engineering
47	Wheeled tractors
48	Motor vehicles
49	Aerospace equipment
50	Other vehicles
51	Engineers' small tools
52	Cutlery and jewellery
53	Bolts, nuts, screws, etc.
54	Wire and wire manufactures
55	Cans and metal boxes
56	Other metal goods
57	Production of man-made fibres
58	Cotton, etc. spinning and weaving
59	Woollen and worsted
60	Hosiery and knitted goods
61	Carpets
62	Household textiles and handkerchiefs
63	Textile finishing
64	Other textiles
65	Leather, leather goods and fur
66	Clothing
67	Footwear
68	Bricks, fireclay and refractory goods
69	Pottery and glass
70	Cement
71	Other building materials, etc.
72	Furniture and bedding, etc.
73	Timber and miscellaneous wood manufactures
74	Paper and board
75	Packaging products of paper, board, etc.
76	Other paper and board products
77	Printing and publishing
78	Rubber
79	Plastic products n.e.s.
80	Other manufacturing
81	Construction
82	Gas
83	Electricity
84	Water supply
85	Railways
86	Road transport
87	Other transport
88	Communication
89	Distributive trades
90	Miscellaneous services

Table 4.1
(cont.)

Numbering of Consumption Sectors
in CSO 1968 Input-Output Tables

CSO No.	Our No.	Sector
93	91	Consumers' Current Expenditure
94	92	Public Authorities' Current Expenditure
95	93	Fixed Capital Formation
96	94	Stocks of Capital
97	95	Exports of Goods and Services

Table 4.2

Reference Numbering of Ecologic Commodities
Used in this Study

Ecologic Commodity	Symbol
1. Particulates	Part.
2. Carbon Monoxide	CO
3. Sulphur Dioxide*	SO_2, SO_x
4. Sulphur Trioxide	SO_3
5. Hydrocarbons	HCO
6. Nitrogen Oxides	NO_x
7. Aldehydes	HCHO
8. Organic Acids	
9. Fluorides	Fluorides
10. Ammonia	Amm., Ammonia
11. Acid Mist (H_2SO_4)	Acid Mist
12. Hydrogen Chloride	HCL

* Where sulphur Oxides are not broken down by EPA into SO_2 and SO_3 then they are here classified as SO_2

4-2 PRIMARY AND SECONDARY ECOLOGIC COEFFICIENTS: METHOD OF CALCULATION

The method adopted for the calculation of ecologic coefficients involves two stages:

(1) The application of modified EPA (etc.) coefficients [3] for pollution to *physical* quantities consumed or produced by industrial and consumption sectors in the U.K. in 1968. These factors are dubbed "primary" ecologic coefficients.

(2) The expression of ecologic magnitudes for the base-year as a proportion of each sector's total or gross output in value terms in that year. Coefficients of this set are entitled "secondary" ecologic coefficients.

Secondary ecologic coefficients are assumed constant over the relevant slice of time (the 'projection period').

4-3 1968 ECOLOGIC MAGNITUDES: FUNDAMENTAL INFORMATION

(i) *Sources of Information on Polluting Materials*

Calculation of ecologic magnitudes for the base-period requires estimates of the physical (tons, gals, etc.) quantities of various commodities used and/or produced. For one of the most important sources of industrial pollution, fuel usage, the input quantities are specified for those sectors covered by the Census of Production in the Report(s) for the sector in question. With respect to those factors requiring the physical quantity *produced* by a sector covered in the Census a different method should ideally be employed to determine the relevant physical quantity. This is because the Census Reports merely contain estimates of the physical quantities *sold* by industries: summation of the physical quantities sold by each industry will not generally yield a figure that equates with the amount actually *produced* by industry; the quantity sold will deviate from this figure by the amount of *intra*-industry sales of the commodity. Where a primary product, such as Aluminium, undergoes a series of manufacturing processes before being sold to final demand (say) and these processes are executed by other firms *in the industry* this 'surplus' over production recorded by the Census will tend to be highest. The obstacle can, however, be overcome. Where independent production totals can be obtained it is easy to make certain plausible assumptions about the relation between sales and production and so distribute this total amongst the industries 'selling' the commodity. In the present work we assume in fact that production is simply proportional to sales. For sectors other than those covered by the Census, including final consumption, an *ad hoc* procedure was adopted.

(ii) *Data on Processes*

One of the most serious data limitations under which the would-be student of the U.K. environment is constrained to work is the absence of accurate and detailed information on *production processes*. The applicability of EPA factors to the U.K. strictly depends on the possibility of identifying identical production processes for commodities produced here and in America. Absence of at least a plausible presumption to this effect would vitiate the empirical analysis to a considerable degree. Such data is not, however, obtainable, from the Census Reports themselves.

Reports of the Alkali Inspectorate contain quite an amount of relevant data on this subject, though no numbers of plants, capacities or usage rates are usually specified, thus compelling assumptions to be made to cover the lacunae. Other utilised sources of information included individual plant managers/ information officers, trade associations, local authorities, public corporations and government departments. In many respects indeed this assistance has been indispensable to achieving a measure of accuracy even though the standards attained fall regrettably far short of the ideal.

(iii) *Treatment of Individual Emissions*

Information in the Reports of the Alkali Inspectorate [4] indicates that differences are to be expected between the abatement efficiencies of processes operated by sectors registered under the Alkali and Works, etc. Acts from those not so registered. Though the average percentage can only be estimated roughly and may vary somewhat from sector to sector we assume in this study on the basis of information in the Reports and correspondence that control is 90% effective over the EPA's "uncontrolled" factors.

Below is a list of sectors governed by the Inspectorate and subject to this assumed control.

Table 4.3

Sectors Registered Under the Alkali Act

15	Coke Ovens, etc.
16	Mineral Oil Refining
17	General Chemicals
25	Other Chemical Industries
26	Iron Castings, etc.
27	Other Iron and Steel
28	Aluminium, etc.
29	Other Non-Ferrous Metals
68	Bricks, etc.
69	Pottery and Glass
70	Cement
82	Gas
83	Electricity

Next is included a list of sources for primary emission factors relating to the "stationary" (as opposed to vehicle or plane) usage of fuels.

Table 4.4

Sources of Primary Emission
Factors for Fuels, "Stationary Sources"

Fuel	EPA Table
Bituminous Coal	1.1-2
Anthracite Coal & Smokeless Fuel	1.2-1
Fuel Oil	1.3-1
Natural Gas	1.4-1

(iv) *Conversion Factors, Sulphur and Ash Contents and Fuel Densities*

Data for these quantities is presented in Tables 4.5 - 4.8 below:

Table 4.5

Conversion Factors Employed in the Study

All figures in the tables used in this study are Imperial quantities unless it is explicitly stated to the contrary. EPA data is in American units (short tons, U.S. Gals, etc.) and some source magnitudes are also expressed in metric units so that conversion is necessary.

1. *Units of Mass*

1 U.S. lb	=	1 Imperial lb
1 lb	=	453.592 gm
1 gm	=	0.002,205 lbs
1 Short ton (2000 lbs)	=	0.892,857 Imperial tons
1 Imp. ton (2240 lbs)	=	1.120 Short tons
1 Long ton (2400 lbs)	=	1.071,428 Imp. tons
1 Imp. ton	=	0.933,334 Long tons
1 Short ton	=	0.907,184 Metric tons
1 Metric ton	=	1.102,312 Short tons
1 Imp. ton	=	1.016,048 Metric tons
1 Metric ton	=	0.984,205 Imp. tons

2. *Units of Volume*

1 Imp. Gallon	=	1.200,950 U.S. Gallons
1 U.S. Gal	=	0.832,674 Imp. Gals
1 U.S. Bbl	=	42 x 0.832,674 Imp. Gals
	=	34.972,308 Imp. Gals

3. *Units of Heat*

1 Btu	=	0.252 Kilocalories (1 Kilocal. = 10 Cals.)
1 Kilocal.	=	3.968,254

Table 4.6

National Survey Estimates of Sulphur Contents of Fuels[1]

		S. Content (% by wt.)
1.	*Petroleum Products*	
	Motor Spirit	0.10
	Derv	0.35
	Gas Oil	0.75
	Fuel Oil (Power Stations)	2.60
	Fuel Oil (Other)	2.60
	Refinery Fuel	1.95
2.	*Coal*	
	Domestic	1.30
	Industry etc.	1.40
	Power Stations	1.53
	Gas Works	0.72
	Coke Ovens	N.A.
3.	*Solid Smokeless Fuel*	
	Domestic	1.00
	Industry, etc.	1.00

---

[1] [5], p.26 . Their industrial classification.

Table 4.7

Ash Contents (% by weight) of Fuels[1]

	Fuel	Ash Content
1.	*Bituminous Coal*	
	Power Stations	16
	Industry etc. + Gas Works	9
	Coke Ovens	7
	Domestic	5[2]
2.	*Anthracite Coal and Smokeless Fuel*	
	Power Stations + Industry etc., + Gas Works + Coke Ovens	3.6
	Domestic	N.A.[2]

---

[1] Source: National Coal Board statistics, priv.comm.

[2] No proportions for *hand-fired* units incorporated in EPA Table

Table 4.8

Fuel Densities

Fuel	Inverse Density[1] Imp.Gals/Imp.ton	Density lbs/Imp.Gal
Derv	267	8.389,513
Gas/Diesel Oil	267	8.389,513
Fuel Oil	235	9.531,915
Motor Spirit	301	7.441,860
Burning Oil (kerosine)	284	7.887,324

Calorific Value of Gas[2]

U.K. weighted average: 499.3 Btu/ft^3

[1] See [6], p.64

[2] British Gov., priv.comm., 25.2.74

4-4 ACTUAL CALCULATIONS

4-4.1 Introduction

1968 ecologic magnitudes are classified into two categories: fuel emissions, from stationary and non-stationary sources; and non-fuel emissions. Those falling into the first category are calculated by determining a set of seven general primary fuels emissions coefficients (ecologic commodities 1 through 7 in Table 4.2), applying to the majority of economic sectors. (For exceptions see 4-3.3 above, and individual data sheets.) Emissions falling into the second category have no common set of primary coefficients. An *ad hoc* computational procedure was employed in this case.

4-4.2 Fuel Emissions: Primary Coefficients

In this section general primary vehicle-emissions factors are calculated. These are applied to the vehicle fuel consumption quantities for the overwhelming majority of industrial sectors in the study. Fuel inputs are estimated in a later section together with ecologic magnitudes (the latter forming the elements of the matrix subsequently referred to as the PF matrix) relating to fuels.

4-4.2a From Stationary Sources

Primary emission factors for stationary sources are presented in Table 4.12: Primary Emission Factors for Fuels in Computable Form. EPA sources for this data is given in Table 4.4, and the sulphur and ash contents of fuels in Table 4.6 and 4.7.

4-4.2b From Mobile Sources

I *Sulphur Dioxide*

Two variables are estimated specifically for the U.K. and used to modify the EPA's emission factor for SO_2 ([3], Table 3.1.2-8 for motor spirit, and Table 3.1.3-1 for derv) to render them applicable here; namely, the sulphur content (S-content) of fuels, and vehicle mileage to the gallon.

Ii *S-Contents*

It is assumed for both motor spirit- and derv-fuelled vehicles that S-contents are proportional to SO_2 emission factors. Since U.K. and U.S. S-contents are known data, EPA emissions coefficients facilitate the calculation of the appropriate U.K. factors. The resulting coefficients are expressed in units of lb/mile. These can be converted to lb/gal if the respective mileages/gal can be estimated. We have, then,

SO_2 Primary Emission Factors (lb/mile)

Motor Spirit Vehicles:	0.000,895
Derv Vehicles:	0.002,431

Iii *Mileages/Gallon*

To obtain estimates of average mileage to the gallon for U.K. vehicles we converted fuel volumes to masses, and, using the coefficients data of Ii, expressed SO_2 emissions for each fuel in units of lb/ton, with mileage/gal as an unknown in this quantity. The resulting expression was then equated to the National Survey estimate of SO_2 per unit emissions (obtained by dividing 1968 SO_2

emissions by the corresponding fuel consumption) of 5.152 lb/ton (motor spirit) and 14.336 lb/ton (derv) respectively. The equations were then solved for vehicle mileages to the gallon. For motor spirit the figure obtained was 19.124,233 mile/gal, and for derv 22.086,747 mile/gal. These figures were then averaged with estimates of mileage to the gallon obtained from independent sources[1], yielding

Mileages to the Gallon

Motor Spirit Vehicles:	22.228,667
Derv Vehicles:	17.025,374

Substitution of these figures into the respective equations discussed above results in our first estimates of SO_2 primary coefficients:

SO_2 Primary Emission Factors (lb/gal)

Motor Spirit Vehicles:	0.019,895
Derv Vehicles:	0.041,389

The coefficient for motor spirit diverges by an amount (+)16% from that calculated on the National Survey data. That for derv shows a discrepancy of (-)23% however, and it was decided to adopt the National Survey estimate of 0.053,693 lb/gal in preference.

---

[1] British Leyland, Scottish Omnibuses, and National Carriers, in priv.comms.

II *Other Pollutant Emissions*

For the remaining significant pollutant emissions from vehicles, viz., Particulates, Carbon Monoxide, Hydrocarbons, and Nitrogen Oxides, no vehicle mileage is specified by the EPA. Factors for the last three of these ecologic commodities are also differentiated with respect to time in the EPA data, thus introducing the American Clean Air legislation as a further variable to be considered in applying them to the U.K.

IIi *Motor Spirit*

The lower mileage/gal of American vehicles is bound to exert some influence on the emissions of these remaining four substances. We might judge that the outputs of pollution should be reduced in proportion to the estimated relative vehicle fuel consumption per mile. The ratio

$$\frac{\text{U.S. Motor Spirit Vehicles-in-General mile/gal}}{\text{U.K. Motor Spirit Vehicles-in-General mile/gal}}$$

is known to be equal to

$$\frac{13.6}{(22.228{,}667)(0.832{,}674)} = 0.734{,}768$$

13.6 being the EPA's figure ([3], Table 3.1.2-8,n.b.) for American mileage/gal, 22.228,667 our estimate of the corresponding U.K. quantity from the previous section, and the other figure a conversion factor from Imperial to U.S. gallons. Though this average figure may be biased by the influence of the private car we assume that it can be applied to scale down industrial vehicle emissions, hoping that in the ratio the discrepancies cancel each other out. The proportion is in any event only applied to ecologic commodities la, 2, 5a and 6 in the Table 4.9, the remainder are assumed unaltered from their pre-1968 values. ([3], Tables 3.1.2-8)

Table 4.9

Ecologic Output Coefficients from
Commercial Petrol-Fuelled Vehicles

Ecologic Commodity	'Natural pre-'68' lb/mile	Reduced 1968 lb/mile	Reduced 1968 lb/gal
1a. Parts.(Exhaust)	0.000,750	0.000,551	0.012,248
1b. Parts.(Tires)	0.000,441	0.000,441	0.009,803
2. Carbon Monoxide	0.239,243	0.175,788	3.907,533
5a. Exh. Hydro.	0.020,727	0.015,230	0.338,543
5b. Crank.Hydro.	0.008,379	0.008,379	0.186,254
5. Total Hydro.	0.029,106	0.023,609	0.524,797
6. Nitrog. Oxides	0.006,064	0.004,456	0.099,051

Notes: (a) An average of the figures EPA gives for different altitudes was adopted.

(b) Row 5 = Row 5a + Row 5b.

(c) Parts. = Particulates
Exh. Hydro. = Exhaust Hydrocarbons
Crank.Hydro. = Crankcase Hydrocarbons
Nitrog. Oxides = Nitrogen Oxides

Table 4.10

Ecologic Output Coefficients
for Derv-Fuelled Vehicles

Ecologic Commodity	'< 1975' lb/mile	'< 1975' lb/gal
1a. Parts.(Exhaust)	0.001,610	0.035,560
1b. Parts.(Tires)	0.000,441	0.009,740
2. Carbon Monoxide	0.003,749	0.082,803
5a. Exh. Hydro	0.000,992	0.021,910
5b. Crank.Hydro	0.008,379	0.185,065
5. Total Hydro.	0.009,371	0.206,975
6. Nitrog. Oxides	0.003,528	0.077,922

III *Motor Spirit and Derv Quantities in the Census*

To apply the coefficients of section II to fuel consumption by industrial sectors ideally requires an exact breakdown of the *Census* item "Derv Fuel and Motor Spirit for Use in Road Vehicles". Such a breakdown is not, however, to be obtained from the *Census* Reports themselves due to the basic limitations of the survey of production. The Department of Trade and Industry's *Digest* ([6] Table 43, provides data on motor spirit and derv fuel consumption and on certain assumptions a 'guesstimate' of the relevant *Census* proportions may be procured.

The proportion of the *Digest's* motor spirit item "Cars and Motor Cycles" applicable to *commercial* vehicles is evidently very small (most commercial vehicles being vans); we assume, rather arbitrarily that it is some 5%. The *Digest's* "Goods Vehicles" category is taken as wholly applicable to *Census* industries both for motor spirit and derv. The proportion of motor spirit in the *Census* fuel quantity is then calculated as

$$\frac{\text{Commercial Motor Spirit}}{\text{Commercial Derv + Commercial Motor Spirit}} = \frac{2934.5}{6438.5}$$

$$= 0.455{,}774 = 46\%$$

Call this proportion r. Then the corresponding proportion for derv is obviously $1 - r$. Thus we have

$$r = 46\%$$

$$1 - r = 54\%$$

(which, as it happens, are roughly the expected proportions for the fuels in the absence of any information with regard to their 'true' values).

To maximise computational efficiency instead of calculating ecologic magnitudes by applying the coefficients of the Tables 4.9 and 4.10 to the *Census* fuel quantities we note the relations

$$x_i . r\Pi_m = P_{mi}$$

$$x_i . (1 - r)\Pi_d = P_{di}$$

where

x_i = the value of "derv fuel and motor spirit for use in road vehicles" for *Census Report* i ;

Π_m, Π_d = the ecologic coefficients (with respect to some pollutant) for motor spirit and derv respectively, as in section II, Tables 4.9 and 4.10;

P_{mi}, P_{di} = the total output of an ecologic commodity from the establishments in *Report* i from motor spirit and derv combustion.

New emission factors are defined for each pollutant as $r\Pi_m$ and $(1 - r)\Pi_d$. These are then applied directly to the quantities x_i. Modified ecologic coefficients are presented in Table 4.11 below.

Table 4.11

Modified Ecologic Output Coefficients
for Commercial Petrol- and Derv-fuelled Vehicles

Ecologic Commodity k	Modified Ecologic Coefficients (lb/gal)	
	Motor Spirit Vehicles	Derv Vehicles
1a	0.005,634	0.019,202
1b	0.004,509	0.005,260
2	1.797,465	0.044,714
3	0.009,152	0.028,994
5	0.241,407	0.111,767
5a	0.155,730	0.011,831
5b	0.085,677	0.099,935
6	0.045,563	0.042,078

Finally, this data is converted to a set of units uniform with the other coefficients obtained from non-stationary sources of emissions. The results are presented in Table 4.12.

Table 4.12

General Primary Emission Factors for Fuels in Computable Form

Ecologic Commodity	Fuel	Emission Factor
1. Particulates	1. Coal a	131,040 lb/10^3 Tons
	2. Coke etc.	38,080 lb/10^3 Tons
	3. {Derv	19.202 lb/10^3 Gals
	Motor Sp.	5.634 lb/10^3 Gals
	4. O.L.F.	22.820 lb/10^3 Gals
	5. Gas (1)	3.004 lb/10^3 Therms
	6. Gas (2)	3.605 "
	7. Gas (3)	3.805 "
	8. Coal b	286,720.0 lb/10^3 Tons
	9. Derv Tires	4.509 lb/10^3 Gals
	10. M.S. Tires	5.260 "
2. Carbon Monoxide	1. Coal a	2,240 lb/10^3 Tons
	2. Coke etc.	3,920 "
	3. {Derv	44.714 lb/10^3 Gals
	Motor Sp.	1,797.465 "
	4. O.L.F.	4.80 "
	5. Gas (1)	3.405 lb/10^3 Therms
	6. Gas (2)	3.405 "
	7. Gas (3)	4.006 "
	8. Coal b	1,120.0 lb/10^3 Tons
	9. Derv Tires	0.0 lb/10^3 Gals
	10. M.S. Tires	0.0 "
3. Sulphur Dioxide	1. Coal a	59,584 lb/10^3 Tons
	2. Coke etc.	42,560 "
	3. {Derv	28.994 lb/10^3 Gal
	Motor Sp.	9.152 "
	4. O.L.F.	466.810 "
	5. Gas (1)	0.120 lb/10^3 Therms
	6. Gas (2)	0.120 "
	7. Gas (3)	0.120 "
	8. Coal b	65,117.0 lb/10^3 Tons
	9. Derv Tires	0.0 lb/10^3 Gals
	10. M.S. Tires	0.0 "
4. Sulphur Trioxide	1. Coal a	0.0 lb/10^3 Tons
	2. Coke etc.	560.0 "
	3. {Derv	0.0 lb/10^3 Gal
	Motor Sp.	0.0 "
	4. O.L.F.	6.25 "
	5. Gas (1)	0.0 lb/10^3 Therms
	6. Gas (2)	0.0 "
	7. Gas (3)	0.0 "
	8. Coal b	0.0 lb/10^3 Tons
	9. Derv Tires	0.0 lb/10^3 Gals
	10. M.S. Tires	0.0 "
		(cont.)

Table 4.12
(cont.)

Ecologic Commodity	Fuel	Emission Factor
5. Hydrocarbons	1. Coal a	2,240 lb/10^3 Tons
	2. Coke etc.	130 "
	3. {Derv	111.767 lb/10^3 Gals
	Motor Sp.	241.407 "
	4. O.L.F.	3.600 "
	5. Gas (1)	0.200 lb/10^3 Therms
	6. Gas (2)	0.601 "
	7. Gas (3)	1.602 "
	8. Coal b	340.0 lb/10^3 Tons
	9. Derv Tires	0.0 lb/10^3 Gals
	10. M.S. Tires	0.0 "
6. Nitrogen Oxides	1. Coal a	16,800 lb/10^3 Tons
	2. Coke etc.	15,960 "
	3. {Derv	42.078 lb/10^3 Gals
	Motor Sp.	45.563 "
	4. O.L.F.	72.060 "
	5. Gas (1)	120.170 lb/10^3 Therms
	6. Gas (2)	46.069 "
	7. Gas (3)	24.033 "
	8. Coal b	20,160.0 lb/10^3 Tons
	9. Derv Tires	0.0 lb/10^3 Gals
	10. M.S. Tires	0.0 "
7. Aldehydes	1. Coal a	6 lb/10^3 Tons
	2. Coke etc.	0.0 "
	3. {Derv	0.0 lb/10^3 Gal
	Motor Sp.	0.0 "
	4. O.L.F.	1.8 "
	5. Gas (1)	0.0 lb/10^3 Therms
	6. Gas (2)	0.0 "
	7. Gas (3)	0.0 "
	8. Coal b	0.0 lb/10^3 Tons
	9. Derv Tires	0.0 lb/10^3 Gals
	10. M.S. Tires	0.0 "

4-4.3 Fuel Emissions: Energy Inputs

4-4.3a Initial Estimates for "Census" Industries

The Census, 1968, provides data on physical imputs of the following fuels to industrial sectors:

1. Coal
2. Coke
3. Derv and Motor spirit
4. Other Liquid Fuels
5. Gas

(6. Electricity)

(See Table 10 of any of the Reports [1]. Emissions from electricity usage are assumed to be zero, since we calculate emissions from the fuels used in the generation of electricity, and attribute these to the relevant public utility (CSO sector 83: Electricity).)

Fuels 1, 2, and 5 do not present any problem: physical inputs are almost invariably specified. But liquid fuels (3 and 4) raise difficulties because frequently a Report will contain a physical entry for a given fuel, a value quantity for that fuel, and a second value quantity for which no physical quantity is specified as counterpart. The procedure adopted in such cases was to estimate the unknown physical quantity by calculating an average cost per unit of fuel from the first pair of quantities (for the same, or related industries, - if no physical quantity at all is presented), and then to divide this into the value of unspecified fuel purchases.

4-4.3b Checking and Adjustment Procedure for Liquid Fuel Estimates to Census Industries

It is desirable to have some independent check on the validity of the Census estimates just discussed. However, there is some difficulty in comparing estimates from different sources due to disparities of classification. Table 44 of the Digest of Energy Statistics[1] is constructed in accordance with the 1958 SIC, but does provide such an independent estimate for liquid fuels (the most problematic subset) into CSO sectors 3-84. The examined correspondence between the 1958 and 1968 SIC's (the latter being used by CSO) is good.

TABLE 4.13

Consumption of Gas/Diesel*
and Fuel Oil by Industry, 1968

Digest Sector	Gas/Diesel		Fuel Oils		Totals
	10^3 Tons	10^3 Gals**	10^3 Tons	10^3 Gals***	10^3 Gals
Manufacturing Industry	2,279		17,853		
Petroleum Industry	62		5,446		
Public Utilities****	575		6,689		
Building & Contracting	629		128		
Mining & Quarrying	293		215		
Total	3,838	1,024,746	30,331	7,127,785	8,152,531

* Includes Derv
** Using a Digest conversion factor of 267 gal/ton
*** Using a Digest conversion factor of 235 gal/ton
**** Excluding Railways

Source: Table 44, Digest of Energy Statistics, 1968-69

Adding the quantity of motor spirit[2] of 731,731 x 10^3 gals, we derive an aggregate total of 8,884,262 x 10^3 gals. This compares with an initial estimated total consumption of the same fuels by the corresponding Census

[1] DTI: Digest, 1968-69

[2] Digest Table 43 under 'Goods Vehicles', using their conversion factor of 301 gal/ton

industries (viz. sectors 3-84) of:

Derv & Motor spirit }
Other liquid fuel } $6,523,768.0 \times 10^3$ Gals

Hypothesising that the Digest figures are not subject to error implies that our census estimates of fuel consumption by this subset of sectors is some 26% too low. Explanation of some of this error can be readily found in the technique of estimating sectorial inputs. Actual unit costs of fuels not specified in physical quantities in the Census Reports are probably lower than those which are specified, e.g. due to lower quality. Hence use of the 'price' estimated from specified quantities and values results in an underestimation of physical quantities for the unspecified elements.

Since the proportion of fuel inputs unspecified varies between industries the device of distributing the total error quantity (viz. $2,360,494 \times 10^3$ gals = $(8,884,262 - 6,523,768) \times 10^3$ gals) in the same average proportion of 26% between industries is inadmissible. An alternative procedure is to hypothesize that individual sectorial errors are proportional to the absolute quantities estimated in those sectors, and to distribute the total error accordingly. This is simply done by calculating the estimate in a sector S as a proportion of the total estimate and then multiplying this proportion into the total error quantity. Thus if no quantity is estimated for sector S it will receive no part of the total error. Because the apportioned error quantities are aggregates of derv, motor spirit and fuel oil, they are divided up into 'derv and motor spirit' and 'other liquid fuels' in accordance with the relative proportions these respective fuels go to make up each sector's fuel estimate. Thus if estimated quantities of 'derv and motor spirit' and 'other liquid fuels' are in the ratio 25:75 for sector S, then it is this ratio which governs the breakdown of the aggregate error quantity to be added to sector S's fuel consumption.

In mathematical terms the procedure just outlined may be expressed as follows:

Let δ_i, λ_i be the 'exact' Census quantities of derv + m.s. and OLF respectively input to sector i $(i=3,84)$;

d_i, l_i be the original estimates of these quantities;

$r_i^d = d_i/f_i$ be the proportion of derv + m.s. in aggregate estimate of fuel input to sector i $(f_i=d_i+l_i)$;

$r_i^l = 1 - r_i^d$ be the corresponding OLF proportion;

$p_i = f_i/\Sigma f_i$ be the proportion of 'total error' (Σf_i) accruing to the ith sector;

$e_i = p_i E$ be the aggregate error quantity to be added to the ith sector, with

$E = F - C = F - \Sigma(\delta_i + \lambda_i + d_i + l_i)$ the total aggregate error over all sectors (F being the Digest's 'exact' liquid fuels input to 'industry').

Then

$$d_i^* = r_i^d e_i \tag{4.1}$$

and

$$l_i^* = r_i^l e_i \tag{4.2}$$

are respectively the estimates of additional derv + m.s. and OLF to be assigned to the ith sector.

We have, by definition,

$$d_i^* = r_i^d e_i$$

$$= \frac{d_i}{f_i} p_i E$$

$$= \frac{d_i}{\Sigma f_i} [F - \Sigma(\phi_i + f_i)]$$

$$\text{with } \phi_i = \delta_i + \lambda_i$$

$$= d_i k \tag{4.3}$$

where

$$k = \frac{F - \Sigma\phi_i}{\Sigma f_i} - 1 \tag{4.4}$$

Final estimates of inputs of derv + m.s. and OLF to sector i may now be written down as

$$\delta_i^* = \delta_i + d_i + d_i^* = \delta_i + (1 + k)d_i \tag{4.5}$$

$$\lambda_i^* = \lambda_i + l_i + l_i^* = \lambda_i + (1 + k)l_i \tag{4.6}$$

Hence

$$\Sigma\delta_i^* = \Sigma\delta_i + (1 + k)\Sigma d_i \tag{4.7}$$

$$\Sigma\lambda_i^* = \Sigma\lambda_i + (1 + k)\Sigma l_i \tag{4.8}$$

and recalling the definition of F :

$$\Sigma(\delta_i^* + \lambda_i^*) = \Sigma\phi_i + (1 + k)\Sigma f_i \tag{4.9}$$

$$= F \tag{4.10}$$

thus providing a check on the accuracy of calculations. Computations for this set of estimates were programmed on a computer using program CORRIGENDUM located in the Appendix.

4-4.3c Initial Estimation of Fuel Inputs to Sectors not Included in Census Set

The classification used by the Digest [5] for liquid fuel inputs to industry and final demand is based on the 1958 Standard Industrial Classification (SIC), and therefore, as CSO Input Output Sectors are defined in terms of the 1968 SIC it was necessary to provide a classification converter between the two. As can be seen from Table 4.14 the correspondence between SIC's is exact in many cases but several minimum List Headings had to be found counterparts in the later classification.

From Table 4.14 below we obtain estimates of liquid fuel inputs to sectors 89 (Distributive Trades), 90 (Miscellaneous Services), 91 (Consumers' Current Expenditure) and 92 (Expenditure by Public Authorities). For the remaining fuels for these four sectors and all fuels for the remaining sectors the reader is referred to individual datasheets (see section 4-4.4).

TABLE 4.14

Classification Convertor for Liquid Fuel Consumption between Digest Table 44 and C.S.O. Sectors[1]

Digest Categories	SIC 1958	SIC 1968 (est.)	CSO SIC 1968 GROUPING	CSO SECTOR 1968	Gas/Diesel		Fuel Oil	
					10^3 Tons	10^3 Gals	10^3 Tons	10^3 Gals
Places of Enter^t.	881, -2, -3	881, -2, -3	861, 862, 864, 865, 866, 871, 873, 876, 879, 881, 883, 884, 885, 886, 889, 892, 893, 894, 895, pts. of 860, 863, 872, 874, 882, 887, 888, 899.	90: Miscell Services	76		96	
Laundries	885	892			59		270	
Catering Ests.	884	884, -5, -6			156		134	
Offices	-	-			238		359	
Other Premises	-	-			94		85	
Subtotals	-	-	-	-	623	166,341	944	221,840
Medical Welfare Ests.	874	874	875, 891, 901, 906, and pts. of 860, 863, 872, 874, 882, 887, 888, 899.	94: Expend. By Public Authorities[2] (our S92)	201		746	
Educational Ests.	872	872			289		535	
British Armed Forces	-	-			80		392	
Nat. Govt. Buildings	901	901			116		613	
Local Govt. Buildings	906	906			143		184	
Religious Premises	875	875			128		32	
Subtotals	-	-	-	-	957	255,519	2,502	587,970

(cont.)

TABLE 4.14 (cont.)

Digest Categories	SIC 1958	SIC 1968 (est.)	CSO SIC 1968 GROUPING	CSO SECTOR 1968	Gas/Diesel		Fuel Oil	
					10^3 Tons	10^3 Gals	10^3 Tons	10^3 Gals
Private Houses	-	-	——	93: Consumers' Current Exp. (Our S91)	467		55	
Other Dwellings	-	-			115		148	
Foreign Armed Forces	-	-			30		46	
Subtotals	-	-	-	-	612	163,404	249	58,515
Distributive Trades	810,-20,-31 -32.	810,-11,-12 -20,-21,-31 -32.	810, 811, 812, 820, 821, 831, 832	89: Distributive Trades	241	64,347	444	104,340

Notes: [1] Cols. 2 & 3 compare 1958 and 1968 SIC classifications; Cols. 4 & 5 the 1968 SIC with CSO sectorial grouping.

[2] The group of SIC Minimum List Headings corresponds to CSO Sector 91, but since this is a 'dummy' sector creating value-added only it was considered preferable to treat pollution from such activities as generated by public authorities' consumption, CSO S94.

4-4.4 Non-Fuel Emissions: Primary Coefficients

In this section calculations of primary ecologic coefficients for non-fuel sources of emissions are calculated, together with those of fuel sources not falling within the ambit of the general coefficients of section 4-4.2. These derived coefficients are then used to estimate 1968 'non-fuel' ecologic magnitudes for the sectors in question and the results constitute a matrix subsequently referred to as the NF matrix.

In each sector dealt with in the following *Basic Datasheets* are distinguished the various sources of emissions. Each such source has a separate subsection; one on data sources for quantities of production or consumption of a particular commodity giving rise to emissions, and for primary emissions factors, and a second for the derived tables of ecologic magnitudes.

Abbreviations used in the text are:

Digest	=	Digest [6]
L.F.	=	Liquid Fuels
O.L.F.	=	Census [1] item: "Other Liquid Fuels"
M.S.	=	Census item: "Motor Spirit"
D.T.	=	Derv Tires
M.T.	=	Motor Spirit Tires
G.F.	=	General Fuels (See Table 4.12)
N.F.	=	Non-Fuels Matrix (of Ecologic Magnitudes)
PF	=	Fuels Matrix (of Ecologic Magnitudes)
Eppr	=	Electrostatic Precipitator

S1: AGRICULTURE

Emissions Source 1: Fuels

1.1 Data Sources

A. Consumption: Digest Table 45 (Liquid Fuels) -

Items: Agriculture: Burning Oil, Vaporising Oil, Gas/Diesel (= Derv + M.S.)
Agriculture: Fuel Oil (= O.L.F.).

Table 28 (Coal) -

Items: Agriculture.

See our Table S1.

B. Emission Factors: G.F.

1.2 Ecologic Magnitudes

Table S1.1 and NF matrix

Emissions Source 2: Open Field Burning

2.1 Data Sources

A. Production: Guardian newspaper (15.9.73) reporting on statement of the National Farmers' Union gives the quantity of straw burnt annually as 3.5×10^6 tons.

B. Emission Factors: E.P.A. Table 2.4-1.

2.2 Ecologic Magnitudes

Table S1.1 and NF.

TABLE S1

Fuel Consumption

Economic Commodity	Units	Quantity
1. Coal	10^3 Tons	200
3. Derv M.S.	10^3 Gals	219,850
4. O.L.F.	"	94,000
9. D.T. 10. M.T.	" "	219,850

TABLE S1.1

Ecologic Magnitudes

Ecologic Commodity	SOURCE 1	SOURCE 2		SECTOR
	Ecologic Mag. Tons	Prim. Ecol. Coeff. lb/Ton	Ecologic Mag. Tons	MAGNITUDE Tons
1. Parts.	16,054.0	19.04	29,750.0	45,804.0
2. CO	181,206.3	112.0	175,000.0	356,206.3
3. SO_2	28,653.3	0.0	0.0	28,653.3
4. SO_3	262.3	0.0	0.0	262.3
5. HCO	35,014.1	22.40	35,000.0	70,014.1
6. NO_x	13,125.7	2.24	3,500.0	16,625.7
7. HCHO	76.1	0.0	0.0	76.1

S2: FORESTRY AND FISHING

Emissions Source 1: Fuels

1.1 Data Sources

A. Consumption: Digest Table 44

Items: Total Marine:Fuel Oil (= O.L.F.);
Total Marine:Gas/Diesel Oil (= Derv).

B. Emission Factors: G.F. excepting Derv, for which 'unadjusted' factors are used from Table 4.10.

1.2 Ecologic Magnitudes

See NF matrix.

S3: COAL MINING

Emissions Source 1: Fuels

1.1 Data Sources

A. Consumption: Census

B. Emission Factors: G.F.

1.2 Ecologic Magnitudes

PF Matrix.

Emissions Source 2: Coal Cleaning

2.1 Data Sources

A. Production: National Coal Board (Priv.Comm., 19.10.73) estimates some 1,200 x 10^3 tons of coal thermally dried in 1968.

B. Emission Factors: E.P.A. Table 8.9-1 (using an average of factors for different driers, since types used by N.C.B. are unknown. 95% control assumed).

2.2 Ecologic Magnitudes

Table S3.

TABLE S3

Particulate Emissions Data
for Thermal Coal Drying

Emission Factor lb/ton	Ecologic Magnitude tons
1.14	610.7

S15: COKE OVENS AND MANUFACTURED FUEL

Emissions Source 1: Fuels

1.1 Data Sources

A. Consumption: Census

B. Emission Factors: G.F. subject to 90% control excepting Derv and M.S.

1.2 Ecologic Magnitudes

PF matrix for derv and M.S.; NF matrix for remainder of fuels.

Emissions Source 2: Metalliferous Coke Manufacture

2.1 Data Sources

A. Production: Quantity of coal charged to carbonization process, and manufactured fuel, from the Census.

B. Emission Factors: E.P.A. Table 7.2-1, our Table S15. N.B. Factors are expressed in the E.P.A. publication as per unit of coal charged to furnaces. Also, E.P.A. factors may underestimate SO_2 production from this source because they are based on an average sulphur content of 0.8% by weight. Industry sulphur contents in 1968 were some 1.4% (See Table 4.6). Factors for Bi-product Coking rather than for Beehive Ovens were applied. On the general validity of the coefficients, it should further be remarked that they were shown to, and accepted by, the British Coke Research Association.

2.2 Ecologic Magnitudes

Table S15 and NF Matrix.

TABLE S15

Emissions Data for
Metallurgical Coke Manufacture

Units: lb/ton & Tons

Type of Operation	Parts	SO_2	CO	HCO	NO_x	Ammonia
Bi-Product Coking						
Unloading	0.448	-	-	-	-	-
Charging	1.680	0.022	0.672	2.800	0.034	0.022
Coking Cycle	0.112	-	0.672	1.680	0.011	0.067
Discharging	0.672	-	0.078	0.022	-	0.112
Quenching	1.008	-	-	-	-	-
Underfiring	-	4.480	-	-	-	-
Ecologic Magnitudes	53,059	60,937	19,248	60,937	609	2,721

S16: MINERAL OIL REFINING, LUBRICATING OILS AND GREASES

Emissions Source 1: Fuels

1.1 Data Sources

A.	Consumption:	Included in process emissions calculations, except derv and M.S.
B.	Emission Factors:	For derv and M.S., G.F.

1.2 Ecologic Magnitudes

PF matrix for derv and M.S.

Emissions Source 2: Sulphur Production

2.1 Data Sources

A.	Production:	Reference [8].[1]
B.	Emission Factors:	EPA Table 5.18-1.[2]

2.2 Ecologic Magnitudes

Table S16.1 and NF Matrix

---

[1] 263.529×10^3 Metric Tonnes of Sulphur was recovered by petroleum refineries in 1968.

[2] EPA factor for the Sulphur Removal Process was chosen, subject to Alkali Inspectorate control of some 80%.

Emissions Source 3: Petroleum Refining

3.1 Data Sources

A. Production: Process input data from a sample survey of refineries.[3] Tables S16.2 and S16.3.

B. Emission Factors: EPA Table 9.1-1.

3.2 Ecologic Magnitudes

Table S16.4.[4]

TABLE S16.1

SO_2 Emissions Data for Sulphur Recovery

Sulphur Removal Process	Controlled Emission Factor (lb/ton 100%-S)	Ecologic Magnitude (Tons)
	0.896	103.7

[3] EPA factors require data on the following nine variables for each of the 22 existing (1968) oil refineries in the U.K. for an exhaustive treatment of emissions (See EPA Tables 9.1-1, and 4.4-1):

1. Boilers and Process Heaters.
2. Fluid Catalytic Cracking Units.
3. Moving-Bed Catalytic Cracking Units.
4. Compressor Internal Combustion Engines.
5. Blowdown Systems.
6. Process Drains.
7. Vacuum Jets.
8. Cooling Towers.
9. Storage Tanks (for evaporation losses).

Data on these variables is not officially published so that recourse had to be made to a questionnaire circulated to the 22 refineries requesting information on their 1968 operations. The basis for obtaining this information was confidential and it is in consequence not possible to present in the following tables capacity figures

[3] (cont.)

[3] (cont.)

(which are officially published) alongside the refinery data; thus refineries are identified by an anonymous number only. Table S16.2 shows that of the 10 replies received many provided incomplete information. Sample sizes for variables ranged from 10 down to five. Combining the data in the Table with refinery capacities a set of linear regressions was run, refinery capacity being the independent variable. The results of these regressions demonstrated that of the six with significant R^2's, viz. those for variables 3, 5, 8, 12, 13 and 18, none of the constant terms were significantly different from zero. These regression equations were then deployed to calculate the values of the six dependent variables for each of the refineries for which no data was available (i.e. no reply received). Variable values over refineries were then summed to provide pollution source magnitudes for the U.K. as a whole. (This procedure meant therefore adding estimated and known quantities for a given variable.) Utilising EPA factors total ecologic outputs could be estimated. These calculations, due to the serious lacunae (resulting in small sample size for the regression coefficients) are thus subject to considerable error.

[4] The total for SO_2 emissions obtained by the described process (n.3) was, by comparison with the National Survey estimate far too large. It is unnecessary to abandon these estimates if we make the assumption that although the absolute magnitudes of the seven pollutants are subject to substantial error, their relative magnitudes are about right. Hypothesising, then, that the structure of pollution is reflected by the relative magnitudes of our calculations we can derive the levels of SO_2 and six pollutants not contained in the National Survey. Each quantity is simply scaled down by the same proportion required to reduce our estimate of SO_2 to the National Survey figure. The results are presented in Table S16.4.

TABLE S16.2

Refinery Data Derived from Questionnaire to Oil Companies Concerning 1968 Operations

Variable / Refinery	Refinery Capacity[1] (10^3 Bbl)	Boilers & Process Heaters (10^3 Bbl oil burned)	Boilers & Process Heaters (10^3 ft^3 gas burned)	Fluid Cat. Crkg. Units (10^3 Bbl feed)	Moving Bed Crkg. Units (10^3 Bbl feed)	Compressor Engines (10^3 ft^3 gas)	Blowdown Systems: Pollution Control (10^3 Bbl)	Blowdown Systems: No Control (10^3 Bbl)
	Y	X_1	X_2	X_3	X_4	X_5	X_6	X_7
1	77,119	4,000	2,260,000	12,000	——[2]	226,000	. .[3]	. .
2	5,761	374,060	73,360,000	——	——	——	——	410
3	18,845	270,000	2,150,000	——	——	——	45,000	——
4	2,089	175	——	——	——	——	——	——
5	623	364	——	——	——	——	——	2.5
6	1,363	80	——	——	——	——	. .	. .
7	19,894	345	290,000	——	——	——	27,500	——
8	15,078	248	36.5	——	——	——	9,855	4,380
9	43,350	. .	. .	——	——	——	43,800	——
10	26,388	187	1,708	——	——	——	——	29,200
SUBTOTAL		649,459	78,061,744	12,000	——	226,000	126,155	33,993

(cont.)

[1] Interpolated from figures in [7]. [2] "——" indicates Zero [3] ". ." indicates Unavailable

TABLE S16.2 (cont.)

Variable / Refinery	Process Drains		Vacuum Jets		Cooling Tower	Crude Storage Tanks		Gas etc. Storage Tanks		Crude Storage Tanks		Gas etc. Storage Tanks	
	Pol. Control 10^3 Bbl	No Control 10^3 Bbl	Pol. Control 10^3 Bbl	No Control 10^3 Bbl	10^3 Gals Wtr.	Fixed Roof Cap. 10^3 Bbl	Floating Roof Cap. 10^3 Bbl	Fixed Roof 10^3 Bbl	Floating Roof 10^3 Bbl	Fixed Roof Thro'put 10^3 Bbl	Floating Roof Thro'put 10^3 Bbl	Fixed Roof Thro'put 10^3 Bbl	Floating Roof Thro'put 10^3 Bbl
	X_8	X_9	X_{10}	X_{11}	X_{12}	X_{13}	X_{14}	X_{15}	X_{16}	X_{17}	X_{18}	X_{19}	X_{20}
1	1,000	12,000	52,000	—	58,900,000	. .	. .	. .	. .	—	73,000	. .	. .
2	—	27,927,700	—	1,666	9,500	907	—	1,278	—	3,475	—	3,108	—
3	1,000	340	. .	. .	28,000	. .	. .	. .	. .	. .	. .	. .	. .
4	—	45,650	. .	. .	47,520	560	—	560	—	838	—	838	—
5	—	1,460	—	9,125	64,240	156	—	120	—	1,095	—	. .	—
6	60	60	1,200	—	35,000	44	4	44	4	313	16	313	16
7	1,000	—	. .	. .	—	. .	. .	. .	. .	. .	. .	. .	. .
8	—	25.6	. .	. .	350,400	. .	. .	. .	. .	. .	. .	. .	. .
9	1,900	1,900	2,833	—	—	—	2,000	2,121	2,186	—	45,233	21,854	21,854
10	720	—	. .	. .	16,128	—	1,886	1,810	29,857	—	375	375	375
SUBTOTAL	5,680	27,989,136	56,033	10,791	59,450,788	1,667	3,890	5,933	32,047	5,721	118,624	26,488	22,245

TABLE S16.3

Estimates of Six Process Variables for Refineries Not Completing Questionnaire

Refinery \ Variable	1968 Capacity (10^3 Bbl)	Fluid Catalytic Cracking Est. X_3	Internal Comp. Engines Est. X_5	Process Drains Est. X_8	Cooling Water Est. X_{12}	Crude Fixed Roof Storage Est. X_{13}	Crude Floating Roof Thro'put Est. X_{18}
11	109,417	14,410.2	271,354.2	2,089.9	70,609,525.5	1,137.9	87,139.7
12	46,179	6,081.8	114,523.9	882.0	29,800,463.2	480.3	36,776.9
13	70,578	9,295.1	175,033.4	1,348.0	45,545,747.8	734.0	56,208.3
14	76,232	10,039.7	189,055.4	1,456.0	49,194,415.4	792.8	60,711.2
15	58,742	7,736.3	145,680.2	1,122.0	37,907,681.1	610.9	46,782.1
16	46,494	6,123.2	115,305.1	888.0	30,003,740.6	483.5	37,027.8
17	10,159	1,337.9	25,194.3	194.0	6,555,856.7	105.6	8,090.6
18	0	0	0	0	0	0	0
19	32,039	4,219.5	79,456.7	611.9	20,675,567.7	333.2	25,515.9
20	12,564	1,654.7	31,158.7	240.0	8,107,863.3	130.7	10,006.0
21	1,256	165.4	3,114.9	24.0	810,528.2	13.1	1,000.3
22	1,246	164.1	3,090.1	23.8	804,074.9	12.9	992.3
23	1,283	169	3,181.8	24.5	827,952.0	13.3	1,021.8
SUBTOTAL		61,396.9	1,156,148.7	8,904.1	300,843,416	4,848.2	371,272.9

TABLE S16.4

Emissions Data for Petroleum Refining

Tons

	Parts	SO_x	HCO	NO_x	Ald.	Amm.
Reduced Totals	14,251	229,321	155,540	49,399	466	110

S17: GENERAL CHEMICALS

Emissions Source 1: Fuels

1.1 Data Sources

A. Consumption: Census. See Worksheet.

B. Emission Factors: General Fuels.

1.2 Ecologic Magnitudes

PF matrix.

Emissions Source 2: Sulphuric Acid Manufacture

2.1 Data Sources

A. Production: Alkali Inspectorate [4], 1968, p.15.[1,3]

B. Emission Factors: EPA Table 5.17-1.[2,4]

2.2 Ecologic Magnitudes

Tables S17.5 and NF Matrix

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[1] 2.995 x 10^3 Tons of H_2SO_4 calculated as monohydrate. The same reference provides figures for a breakdown by process:

Table S17.1 Sulphuric Acid:Production By Process.

Process	% of Production
Chamber Process Tower Process	} 4.2
Contact Process	95.8
Total	100.0

[2] E.P.A. provides emission factors only for the Contact Process, but, since this greatly preponderates on both sides of the Atlantic the total ecologic impact is little affected.

[3] (cont.)

(cont.)

[3] Inputs to the Contact Process are specified by the A.I. as follows.

Table S17.2 Sulphuric Acid:Material Inputs to Processes.

Input Material for Sulphur Dioxide	No. of Units in Contact Process	Percentage of Total
1. Sulphur	37	54.41
2. Pyrites ("Sulphide Ores")	9	13.24
3. Anhydrite	10	14.71
3. Spent Oxide	4	5.88
3. Miscell.	8	11.76
Total	68	100.00

[4] E.P.A. factors presuppose an estimate of the strength of the acid produced as well as the material feedstock. For the former, an average of the ranges presented by E.P.A. is used. A conversion chart for feedstocks to facilitate transition from E.P.A. to A.I. classifications is presented below.

Table S17.3 Feedstock Conversion Chart.

Alkali Insp. Class$^{n.}$	E.P.A. Class$^{n.}$
1. Sulphur	1. Recovered Sulphur Bright Virgin " Dark " "
2. Pyrites	2. Sulphide Ores
3. Spent Oxide Anhydrite Miscell.	3. Spent Acid

(N.B. Bracketing in E.P.A. Class$^{n.}$ indicates a simple average of emissions was adopted.
Assuming the proportions in Table S 17.2 represent proportions of 1968 H_2SO_4 production by process, we have:

Table S17.4 Production Feedstock and Acid Mist Emissions.

Feedstock	Per Cent. of Prod$^{n.}$	Quantity 10^3 Tons	Uncontrolled Emissions lb/Ton	Controlled Emissions lb/Ton
1. Sulphur	54.41	1.630	2.085	0.042[2]
2. Pyrites	13.24	0.397	4.816	0.096[2]
3. Anhydrite Spent Oxide Miscell.	32.35	0.969	2.774	2.774
Total	100.00	2.996[1]	N.Appl.	N.Appl.

[1] Not equal to production figure of n.1 due to rounding.
[2] Control eff. 98%.

S17: GENERAL CHEMICALS

Emissions Source 3: Hydrochloric Acid Manufacture

3.1 Data Sources

A. Production: Alkali Inspectorate, Report [4], 1969, (p.12).[5]

B. Emission Factors: E.P.A. Table S7-1.[6]

3.2 Ecologic Magnitudes

Table S17.5 and NF Matrix

TABLE S17.5

Ecologic Data for Hydrochloric and Sulphuric Acid Manufacture

Process	Ecologic Commodity	Emission Factor lb/Ton	Ecologic Magnitude Tons
HCL	Hydrogen Chloride	0.22	5.5
H_2SO_4	SO_2	30.24	40.4
H_2SO_4	Acid Mist	(1) 0.042	0.1
"	"	(2) 0.096	0.1
"	"	(3) 2.774	3.7
	Total Acid Mist	-	3.9

---

[5] 56.134 x 10^3 Tons of 97% HCL.

[6] The table gives emission factors with and without a 'final scrubber'. Since in 1969 a U.K. plant was actually closed down because 'it was operated without a suitable scrubber' (Alkali Insp., *op.cit.*), it can be expected that at minimum the EPA's final scrubber control efficiency obtains in U.K. plants. It is assumed that (in the absence of information on the point) that the production method used is bi-product Hydrogen Chloride.

Emission factor: 0.22 lb/ton acid.

S24: FERTILISERS

Emissions Source 1: Fuels

1.1 Data Sources

A. Consumption: Census.

B. Emission Factors: General Fuels.

1.2 Ecologic Magnitudes

PF Matrix.

Emissions Source 2: Phosphate Fertiliser Production

2.1 Data Sources

A. Production: Monthly Digest of Statistics [12].[1]

B. Emission Factors: EPA Table 6.10-1.[2]

2.2 Ecologic Magnitudes

See Table S24.3.

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[1] 67×10^3 Tons.

[2] Assuming the EPA-stated prevalence of control by Wet Scrubber and EPA collection efficiency. This is corroborated by Alkali Insp. 1969, p.15. Efficiency of 98% adopted. See Table S24.1.

Table S24.1 Emission Factors for Manufacture of Phosphate Fertilisers.

Emission Source	Parts lb/ton	Fluorides[1]
Single Super Phosphate		
Grinding, drying	0.2016	-
Main Stock	-	0.1680
Total	0.2016	0.1680

[1] Both gaseous and particulate; control already assumed in EPA figure.

Emissions Source 3: Nitrogenous Fertilisers

3.1 Data Sources

A. Production: Monthly Digest of Statistics, [12].[3]

B. Emission Factors: EPA Table 6.8-1.[4]

3.2 Ecologic Magnitudes

Table S24.3.

TABLE S24.3

Ecologic Magnitudes from Fertilisers

Process \ Ecologic Comm.	Parts Tons	NO_x Tons	Ammonia Tons	Fluorides Tons
Phosphate Fert.	6.0	-	-	5.0
Nitrogenous Fert.	175.7	183.5	948.9	-
Total	181.7	183.5	948.9	5.0

[3] 874.7 x 10^3 Tons.

[4] From A.I. Reports it is gleaned that Nitrate Fertiliser production is probably at least in part produced by granulators as opposed to grilling towers. EPA factors for the former are in any event adopted, subject to 96% control on driers and 70% on granulators. See Table S24.2.

Table S24.2 Emission Factors for Nitrogen Fertiliser Manufacture.

Process	Particulate lb/ton	Nitr. Oxides lb/ton	Ammonia lb/ton
With Granulator			
Neutraliser[1]	-	-	2.20
Granulator	0.14	0.33	0.17
Dryers & Coolers	0.31	0.14	0.06
Total	0.45	0.47	2.43

[1] Controlled factor based on 95% recovery in recycle scrubber (EPA)

S26: IRON CASTINGS ETC.

Emissions Source 1: Fuels

1.1 Data Sources

A. Consumption: Census

B. Emission Factors: G.F., controlled, except derv. and M.S.[1]

1.2 Ecologic Magnitudes

See PF matrix

Emissions Source 2: Secondary Iron Processing

2.1 Data Sources

A. Production: Amounts charged in furnaces from British Steel [18]; proportions charged in each type of furnace from British Cast Iron Research Association (B.C.I.R.A.)[2].

B. Emission Factors: E.P.A. Table 7.10-1, for uncontrolled Cupola emissions, and B.C.I.R.A.[3] for control efficiencies.

2.2 Ecologic Magnitudes

See Tables S26.1, S26.2.

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[1] Registered under the Akali Acts, implying good control, here assumed 90% efficient.

[2] Priv.Comm., 9.8.73. Statistics provided are:

	%
Cupola:	88
Reverberatory and Rotary:	5
Electric Induction:	7
Total	100

[3] Only percentages by number of installations subject to control were obtainable from B.C.I.R.A.:

(i) Wet Cap. ~ 55%; 50-60% efficiency
(ii) Impingement Scrubber: ~ 35%; 20-30% efficiency
(iii) High Energy Scrubber
(iv) Electrostatic ppr. } ~ 3%; 90% (+) efficiency
(v) Baghouse
(vi) Uncontrolled: ~ 7%
(cont.)

TABLE S26.1

Emission Factors for Secondary Iron Processing

		Parts. lb/ton	Efficiency %
High Energy Scrubbers Electrostatic pprs. Baghouse	'Method A'	1.9040	90
Wet Cap Impingement Scrubber	'Method B'	11.424	40

TABLE S26.2

Particulate Emissions from Secondary Iron Processing

Control Method	Particulates Tons
Method A	5,023.4
Method B	30,140.5
Total	35,163.9

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[3] (cont.)
However, it was stressed that the 7% of cupolas with no control were 'not very important' re production as they tended to be 'small, infrequently used furnaces'; and that, conversely, controls (iii) to (v) were 'very important, since they are in general large plants used for long periods'. To simplify matters, it was assumed all production is by cupola and control is as indicated in the B.C.I.R.A. table above, but with the 7% of 'uncontrolled' production distributed in (i) to (vi) in proportion to 'existing' production estimates, i.e.

(i) 59% production; 50-60% efficiency
(ii) 38% production; 20-30% efficiency
(iii), (iv), (v) 3% production; 90% efficiency

100%

S27: OTHER IRON AND STEEL

Emissions Source 1: Fuels

1.1 Data Sources

A. Consumption: Census.

B. Emission Factors: General fuels, controlled except for M.S. and derv.[1]

1.2 Ecologic Magnitudes

See PF Matrix.

Emissions Source 2: Pig-Iron Production

2.1 Data Sources

A. Production: British Steel Corporation [18], p.27.[2]

B. Emission Factors: E.P.A. Table 7.5-1.[3] See Table S27.1.

2.2 Ecologic Magnitudes

See Table S27.12.

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[1] Fuel emissions from stationary sources (viz. non-motor spirit and derv, in this case) reduced by a factor of 0.01 since industry registered under Alk. Act.

[2] $16,431.7 \times 10^3$ Tons.

[3] Blast furnace controls: wet scrubbers, venturi scrubbers and Epprs acting in series. Efficiency: 99%.
Sintering controls: dry cyclone and wet scrubber/Eppr in series on windbox operations. Efficiency: 99.5% (particulates) and carbon monoxide assumed (after E.P.A.) to be reduced to undetectable levels on Blast furnace and sintering operations, whilst remaining polutants unaffected on discharge operations. See Table S27.1.

Emissions Source 3: Steel Mill Production

3.1 Data Sources

A.	Production:	Crude Steel;[4] BSC [18]. See Tables S27.2, S27.3, S27.5, S27.6, S27.8.
B.	Emission Factors:	E.P.A. Table 7.5-1.[5] See Tables S27.4, S27.7, S27.9, and S27.12 for summary.

3.2 Ecologic Magnitudes

See Table S27.12.

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[4] It is assumed E.P.A.'s division of production into Mills and Foundries corresponds respectively to B.S.C.'s classes of Crude, and Finished and Semi-Finished steel. This was corroborated in correspondence with B.S.C.
Proportions of production by furnace type given by B.S.C. (priv.comm.) in Table S27.2.

[5] My conjecture from information contained in G.E. Speight [16], is that 15% of Open Hearth furnaces have oxygen-lancing and all Open Hearth furnaces are subject to control by Epprs. See Table S27.3.
Electric Arc furnace data is derived from P.A. Matthews [17]. See Table S27.5. The assumptions underlying this Table's construction are: that mean capacity represents all furnaces in a given size class (col.2); that each furnace may be converted to a common 5-ton unit (col.3); that numbers of furnaces operating times the equivalents expressed as a proportion of total may be taken as proxy for annual production percentages by furnace class (col.6), process and control (cols.8,10 and 12). Thus it becomes possible to calculate electric arc production by process and control (cols.13-15). It is assumed that only furnaces ⩾20 Tons are controlled re emissions to the air. See Table S27.6. By a similar logic to that employed in the construction of Table S27.5 we may draw up a table (See S27.6) showing the breakdown of production within control categories (where different factors apply). In accordance with Table S27.5 it is hypothesised that furnaces use O_2 for firing. Emissions coefficients are estimated from those given by E.P.A. for electric arc furnaces, conditioned by oxygen lancing. Since E.P.A. factors for processes subject to Epprs do not distinguish the 'wet' and 'dry' varieties I adopt the E.P.A. figure for control by venturi scrubbers (somewhat more efficient than dry Epprs). See Table S27.7.
Basic Oxygen furnaces: Speight (*op.cit*) does not provide data for the separate or serial efficiency of Epprs or spray chambers - control methods used on Basic Oxygen furnaces; nor is the proportion of production subject to each type of control enunciated. The writer mentions that formerly the situation with respect to control was that these two

[5] (cont.)

(cont.)
techniques were not used serially. He goes on to describe 'the most recent [1971?]' method of fume arrestment by means of limited combustion. Finally, a table is presented showing that all but four of the 20 L.D. (Basic Oxygen) converters used in the U.K. in 1971 were subject to full combustion of flue gas, and nine of those 16 were subject to auxiliary firing. The remaining four had limited combustion in conjunction with wet washing. Hypothesising identical output per converter Table S27.8 is calculated. Emission factors are presented in Table S27.9. With regard to the basic oxygen process the hypothesis utilised is that the control technique of wet washing of flue gases when full combustion is employed generates an emission coefficient identical with the one specified by EPA for spray chambers, and that this and precipitator-governed emissions are unaltered by the practice of heat recovery. This assumption covers the output of particulates from unlimited combustion; from limited combustion systems, i.e. where fume control systems are in use, the coefficient is taken as a 97% reduction on EPA's factor for wet washed gases (for example, second row col.4 of Table S27.9 is got from first row, col.1 x 0.03).

Emissions Source 4: Steel Foundries

4.1 Data Sources

A. Production: Finished and Semi-Finished Steel from BSC Ann.Stats, 1968, Tables 60 and 62.[6] See Table S27.10.

B. Emission Factors: EPA Table 7.13-1.[7] See Table S27.11 and S27.12.

4.2 Ecologic Magnitudes

See Table S27.12.

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[6] Data for production by furnace type is not available in the published literature. By the nature of the emission factors (expressed as per unit of metal charged) the quantity of metal input to furnaces should ideally be employed. Since the input-output ratio is unknown we adopt the output quantity as proxy. Furnace proportions are given in Table S27.10. They were arrived at using the following assumptions: electric induction charge is taken equal to the average of the proportion of iron produced in foundry furnaces of this category and the proportion of crude steel produced in electric induction furnaces in 1969. The remaining percentage is then distributed amongst electric arc and open hearth furnaces pro rata as these species of production bear to one another in the output of crude steel. Applied to the total foundry output these proportions yield the quantity breakdown of Table S27.10.

[7] The form of control hypothesised is (dry) Eppr, except on electric induction furnaces which, according to EPA is rarely governed by collectors of any kind. Here the EPA's 'uncontrolled' factors are taken as appropriate. The use of Epprs reduces emissions from electric arc furnaces by 95% on average, and from open hearth by some 96%.

TABLE S27.1

Emissions from Pig-iron Production

Type of Operation	Total Particulates lb/ton	Carbon Monoxide lb/ton
1. Blast furnaces	1.68	-
2. Sintering		
(a) Windbox	0.045	-
(b) Discharge	0.123	49.28
Total	1.848	49.28

TABLE S27.2

Breakdown of Steelmill Production by Furnace Type

	Per Cent	10^3 Tons
Basic Open Hearth	55	14,224.210
Electric Arc	28	7,241.416
Basic Oxygen	17	4,396.574
Total	100	25,862.200

Source: BSC priv.comm. and Annual Stats. 1968.

TABLE S27.3

Steel Mills: Open Hearth Production by Process and Control

Furnace Type	Per Cent	10^3 Tons
No Oxygen Lance, Eppr.	85	12,090.578
Oxygen Lance, Eppr.	15	2,133.632
Total	100	14,224.210

Source: Proportions conjectured from [16].

TABLE S27.4

Steel Mills: Ecologic Output Coefficients
for Open Hearth Furnaces

Units: lb/ton

Furnace Type	Parts	Fluorides		
		Gaseous	Parts	Total
No Lance, Eppr.	0.19	0.112	0.0007	0.1127
Lance, Eppr.	0.39	0.112	0.0007	0.1127

TABLE S27.5

Electric Arc Production in Steel Mills:[1] By Fuel Type, Lancing, & Control

1	2	3	4	5	6	7	8	
Capacity: Tons	Capacity Average	5-Ton Equivalent of each Furnace	Furnaces Operating	Furnace Equivalent	Proportions of Production by Furnace Size	No. using O_2 for Refining	No. of 5-Ton Equivs. using O_2	
				(3x4)			(3x7)	% of Total*
< 10	5	1	11	11	1.8	9	9	1.5
10-29	20	4	13	52	8.6	13	52	8.6
30-59	45	9	9	81	13.4	7	63	10.4
> 60	115	23	20	460	76.2	20	460	76.2
Total	-	-	53	604	100.0	49	584	96.7

9	10		11	12		13	14	15
No. using Oxy-fuel for Fumeless Relining	No. of 5-Ton Equivs. using Oxy.		No. fitted with Cleaning Equipment	No. of 5-Ton Equivs. fitted with Cleaning		Production using O_2	Production using Oxy fuel	Production Subject to Cleaning
	(3x9)	% of Total*		(3x11)	% of Total*	(Tons x 10^3)	(Tons x 10^3)	(Tons x 10^3)
3	3	0.5	0	0	0	108.021	50.690	0
2	8	1.3	8	32	5.3	622.762	137.587	383.795
3	27	4.5	4	36	6.0	753.107	463.451	434.485
0	0	0	20	460	76.2	5517.959	0	5517.959
8	38	6.3	32	528	87.5	7002.449	651.727	6336.239

* Total from col.5

[1] As of Dec.1968

TABLE S27.6

Steel Mills: Electric Arc Furnaces - Numbers, Production and Control

Furnace Capacity (Tons)	Cleaning System (No.'s)					Total
	Wet Eppr	Dry Eppr	Wet Scrubber	Bag Filter	Fumeless Relining	
80-150 (115)	14	4	-	-	-	18
15-60 (37.5)	1	-	7	8	5	21
Production Totals (Tons)	3630.665	1013.798	589.270	677.978	424.528	6336.239

Note: Bracketed figures indicate assumed average capacity. Production figures obtained by reducing all to common 37.5 -Ton unit. See also Table S27.5

TABLE S27.7

Steel Mills: Ecologic Output Coefficients for Electric Arc Furnaces with Oxygen-lancing

Ecologic Commodity (lbs/ton)		Cleaning System				
		Wet Eppr	Dry Eppr	Wet Scrubber	Bag Filter	Fumeless Relining
Total Particulates		0.246	0.68	0.246	4.92	0.68
Carbon Monoxide		20.16	20.16	20.16	20.16	20.16
Flurorides	Gas.	0.002	0.013	0.002	0.013	0.013
	Part.	0.012	0.012	0.012	0.012	0.012
	Total	0.014	0.025	0.014	0.025	0.025

TABLE 27.8

Steel Mills: LD Converters by Furnace Type, & Control

Condition of Waste Gas	Heat Recovery with Auxiliary Firing		No Heat Recovery		Total
	Wet Washer	Dry Eppr	Wet Washer	Dry Eppr	
Full Combustion (No. of Furnaces)	3	6	-	7	16
Limited Combustion (No. of Furnaces)	-	-	4	-	4
Proportion of Production by Control	15.0	30.0	20.0	35.0	100.0
Production Quantities					
Full Combustion	659.486	1318.972	-	1538.801	3517.259
Limited Combustion	-	-	879.315	-	879.315
Total	659.486	1318.972	879.315	1538.801	4396.574

TABLE S27.9

Steel Mills: Ecologic Output Coefficients for LD Converters

Condition of Waste Gas	Heat Recovery + Auxiliary Firing		No Heat Recovery		Ecologic Commodity (lb/ton)
	Wet Washer	Dry Eppr	Wet Washer	Dry Eppr	
Full Combustion	17.14	0.57	..	0.57	Total Particulates
Limited Combustion	..	..	0.51	..	
Full Combustion	0.00	0.00	..	0.00	Gaseous Flurorides
Limited Combustion	..	..	0.00	..	
Full Combustion	0.067	0.002	..	0.002	Particulate Flurorides
Limited Combustion	..	..	0.002	..	

Note: .. indicates zero production in Table S27.9; hence the coefficient is omitted.

TABLE S27.10

Steel Foundries: Breakdown of Production by Furnace Type

Furnace Type	Per Cent	Quantity of Metal Charged (10^3 Tons)
Electric Ind.	4.00	1,123.5
Electric Arc	32.49	9,126.1
Open Hearth	63.51	17,839.3
Total	100.00	28,088.9

Source: Total production BSC Ann.Stats.

TABLE S27.11

Steel Foundries: Ecologic Output Coefficients from Charging by Furnace Type

Type of Process	Particulates (lb/ton)	Nitrogen Oxides (lb/ton)
Melting		
Electric Arc	0.728	0.011
Open Hearth (ox. lance)	0.448	-
Electric Induction	0.112	-

TABLE S27.12 SUMMARY TABLE

Ecologic Outputs from Iron & Steel (excl. Iron Foundries) by Process

Process	Source Magnitudes 10^3 Tons	Particulates		Carbon Monoxide		Nitrogen Oxides		Fluorides[1]	
		lb/Ton	Tons	lb/Ton	Tons	lb/Ton	Tons	lb/Ton	Tons
1. PIG IRON	16,431.7	1.68	12,323.7	-	-	-	-	-	-
2. STEEL MILLS									
(a) Open Hearth									
(i) No lance, eppr	12,090.6	0.19	1,025.5	-	-	-	-	0.1127	604.5
(ii) Lance, eppr	2,133.6	0.39	371.5	-	-	-	-	0.1127	106.7
(b) Electric Arc									
(i) W. Eppr	3,630.7	0.246	398.7	20.16	32,676.3	-	-	0.014	22.7
(ii) D. Eppr	1,013.8	0.68	307.8	20.16	9,124.2	-	-	0.025	11.3
(iii) W. Scrubber	589.3	0.246	64.7	20.16	5,303.7	-	-	0.014	3.7
(iv) Bag Filter	678.0	4.920	1,489.2	20.16	6,102.0	-	-	0.025	7.6
(v) Fumeless	424.5	0.680	128.9	20.16	3,820.5	-	-	0.025	4.7
(vi) Uncontrolled	905.2	12.320	4,978.5	20.16	8,146.6	-	-	0.280	113.1
(c) Basic Oxygen									
(A) Full Combustion									
(i) W. Washer	659.5	17.140	5,046.3	-	-	-	-	-	-
(ii) D. Eppr	2,857.8	0.570	727.0	-	-	-	-	-	-
(B) Limited Combustion									
(i) W. Washer	879.3	0.510	200.2	-	-	-	-	-	-
(ii) D. Eppr	0.0	-	-	-	-	-	-	-	-
3. STEEL FOUNDRIES									
(a) Electric Arc	1,123.5	0.728	365.1	-	-	0.011	5.5	-	-
(b) Open Hearth	9,126.1	0.448	1,825.2	-	-	-	-	-	-
(c) Electric Induction	17,839.3	0.112	892.0	-	-	-	-	-	-
TOTALS	. .	. .	30,144.3	. .	65,173.3	. . .	5.5	. .	874.3

[1] Total (gaseous + particulate) fluorides.

S28: ALUMINIUM AND ALUMINIUM ALLOYS

Emissions Source 1: Fuels

1.1 Data Sources

A. Consumption: Census

B. Emission Factors: General Fuels, controlled.[1]

1.2 Ecologic Magnitudes

See PF Matrix

Emissions Source 2: Primary Aluminium Smelting

2.1 Data Sources

A. Production: There were no primary Aluminium smelters (in the E.P.A.'s sense of the term) in the U.K. in 1968.[2]

Emissions Source 3: Secondary Aluminium Smelting

3.1 Data Sources

A. Production: Census Report 47, Table 10.
I estimate the total quantity of metal processed to be some 827.63 x 10^3 Tons.

Alkali Inspectorate Reports were used to estimate proportions of production by process, assuming the proportion of furnaces represents this information.[3]

B. Emission Factors: E.P.A. Table 7.8-1, subject to control by Baghouse.[4] Units: per weight of metal processed.

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[1] 90% control assumed on stationary sources.

[2] Information from Non-Ferrous Metals Association (priv.comm.).

[3] These proportions are:

Rotary	% 25.405
Winget	6.486
Reverb.	63.784
Electric Melting	4.325
	100.000

[4] E.P.A. "Reverberatory Furnace" is taken as corresponding to the Alkali Inspectorate's "reverberatory"; all other furnaces in the (cont.)

3.2 Ecologic Magnitudes

See Table S28.1

TABLE S28.1

Emissions Data for Secondary Aluminium Smelting

Furnace Type	Part. Emission Factor lb/ton	Ecologic Output tons
Swarf Degreasing	3.696	112.1
Rotary	0.728	68.3
Winget	0.728	17.4
Reverberatory	1.456	343.1
Electric Melting	0.728	11.6
Total	-	552.5

---

[4] (cont.)

Inspectorate's Report are given average emissions of E.P.A. smelting furnaces.

Swarf Degreasing Furnaces are understood to be what E.P.A. calls "Sweating Furnaces", i.e. 'furnaces to treat dirty scrap in preparation for smelting'. Since this is a process additional to smelting an estimate of the proportion of total production employing this process is required. Now, each of the 185 (non-degreasing) furnaces is assumed to have produced 1/185 of total production; I take 15/185 or 8.1% to be subject to degreasing, after conferring with NFMA (*op cit.*).

S29: OTHER NON-FERROUS METALS

Emissions Source 1: Fuels

1.1 Data Sources

A. Consumption: Census

B. Emission Factors: General Fuels, reduced, except Derv and M.S.

1.2 Ecologic Magnitudes

See PF matrix.

Emissions Source 2: Copper Smelting

2.1 Data Sources

A. Production: World Bureau of Metal Statistics [31].[1] Proportions of production by furnace type estimated from 1968 Report of the Alkali Inspectorate [4] data.[2]

---

[1] 1102.7×10^3 tons

[2] The Alkali Inspectorate [4] provide data (a) for England and Wales only, and (b) for the number of furnaces in each class only. On p.27 is data for the following table.

Table 29.1

Type of Furnace	No. in 1968	No. with Arrestor
Cupola	5	4
Electric Arc	6	3
Reverberatory	29	15
Sklenar	31	2
Electric Induction	195	55
Miscellaneous	61	Not stated
Special Alloys	39	"
Totals	366	79

Column 2 data is taken to reflect, when expressed in % terms, the proportion of production by each furnace type for U.K.

B. Emission Factors: E.P.A. Table 7.9-1.[3] Units: per weight of metal charged.[4] Control method assumed in the baghouse filter, efficiency 95-99.6% (average = 97.3%).[5]

2.2 Ecologic Magnitudes

See Table S29.2

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[3] Strictly, E.P.A. factors apply only to copper alloy production. However, since omission would tend to an underestimation of emissions the inclusion of this process was decided upon. Good control is, in any event, assumed applicable to all production.

[4] Metal produced is here used as proxy since the input-output ratio is not known. This would tend therefore to underestimate total emissions from this source.

[5] E.P.A. states that 'the only air pollution equipment that is generally accepted . . . is the baghouse filter' and gives the stated efficiency range. The use of a bag filter is corroborated by the Alkali Inspectorate. However, the latter lists four other types of furnaces than those, viz. cupola, electric induction, and reverberatory, for which E.P.A. gives emission factors. A simple average of all the E.P.A. emission factors is taken as applicable to the A.I's four remaining furnace types. Column 2, data of Table S29.1, expressed as percentages of the figures in column 1 is taken to represent the percentage of production within each furnace type that is subject to emissions control.

TABLE S29.2

COPPER AND COPPER ALLOYS:

Total Non-Fuel Ecologic Outputs

	1		2		3	4	5	6	7	
Type of Furnace	Total 1968		With Arrestor		Production[2] by Furnace 10^3 Tons	Production 10^3 Tons		Part. Emission Factor lb/ton		Total Part. Emissions
	No.	%	No.	%		Uncontrolled	Controlled	Uncontrolled	Controlled	Tons
Cupola[1]	5	(1.366)	4	(80.000)	15.060	3.012	12.048	81.760	2.208	121.8
Electric Arc	6	(1.639)	3	(50.000)	18.070	9.035	9.035	43.867	1.184	181.7
Reverberative[1]	29	(7.923)	15	(51.724)	87.351	42.191	45.181	78.400	2.117	1,519.4
Sklenar	31	(8.470)	2	(6.452)	93.382	87.312	6.025	43.867	1.184	1,713.1
Electric Ind.[1]	195	(53.279)	55	(28.205)	587.401	421.754	165.676	2.240	0.060	426.2
Miscellaneous	61	(16.667)	.	.	183.754	144.091	39.663	43.867	1.184	2,842.8
Special Alloys	39	(10.656)	.	.	117.482	92.124	25.358	43.867	1.184	1,817.5
Totals	366	(100.000)	79	(21.585)	1,102.5	799.519	302.986	N.App.	N.App.	8,622.5
Derivation of Column:	Datum		(2a/1a)		1b x Total Production	(1b-2b) x 3	2b x 3	Datum	Datum	(4 x 6) + (5 x 7)

[1] Corresponding directly to E.P.A. classification

[2] In 10^3 Tons metal charged

S29: OTHER NON-FERROUS METALS (cont.)

Emissions Source 3: Primary Lead Smelting

3.1 Data Sources

A. Production: We assume that lead bullion is classified (by the E.P.A.) as secondary lead. As no concentrates of lead were produced in U.K. in 1968[6] we conclude no primary lead was produced in that year.

Emissions Source 4: Secondary Lead Smelting

4.1 Data Sources

A. Production: WBMS, *op.cit.*[7]

B. Emission Factors: E.P.A. Table 7.11-1.[8] Controls assumed[9]. Units: per ton processed.[10]

4.2 Ecologic Magnitudes

See Table S29.4

TABLE S29.4

Emissions Data for Secondary Lead Smelting

Ecologic Commodity	Emission Factor lb/ton	Ecologic Magnitude Tons
Particulates	1.092	128.4
Sulphur Oxides	6.552	770.2

[6] See WBMS, *op.cit.*

[7] WBMS provides the following table:
Table S29.3

	Secondary Lead Production 1968 10^3 Tons
English Refined Lead	141.360
Lead Refined from Imported Bullion	90.570
Lead Bullion	31.386
Total	263.316

[8] (cont.)

S29: OTHER NON-FERROURS METALS (cont.)

Emissions Source 5: Primary Zinc Smelting

5.1 Data Sources

A. Production: WBMS, *op.cit.*, and A.M. & S. (Europe) Ltd.[11]

B. Emissions Factors: E.P.A. 7.7-1, and Alkali Inspectorate Reports for controls.[12]

5.2 Ecologic Magnitudes

See Table S29.5

TABLE S.29.5

Ecologic Data for Primary Zinc Processing

Process	Parts. Factor	Parts. Emission
Units	lb/ton	Tons
Roasting	-	-
Sintering	10.08	45.8
Horiz. Retorts		-
Vert. Retorts	11.20	189.9
Electrolyt. Proc.		-
Totals	. .	235.7

[8] Emission factors in this table are process-specific; but the kinds of process used in U.K. are not generally known. We adopt the simple expedient of an arithmetic average of the E.P.A.'s controlled factors, since control is certain to exist, the industry being registered under the Alkali Acts.

[9] Control is here taken to mean hooding followed by baghouse (or equivalent).

[10] Production is taken as surrogate for material processed, thus tending to underestimation of ecologic outputs.

[11] I.D. McDermid of A.M. & S. (priv.comm. 13.8.73) states that 'some 27% [of the 140.625 x 10^3 Tons produced in 1968] was produced by the Vertical Retort process, the balance by IMF furnaces', and goes on to remark that no metal was produced by the Electrolytic or Horizontal Distillation processes, and that it is unlikely that any was roasted or smelted. We assume that IMF furnaces, described by this source as employed for the sintering of lead and zinc concentrates for charge feed, account for 73% of primary zinc output, the remaining 27% being processed in (the E.P.A.'s) Vertical Retorts.

[12] The A.I. does not, to my knowledge, indicate the pervasiveness or (cont.)

S29: OTHER NON-FERROUS METALS (cont.)

Emissions Source 6: Secondary Magnesium Smelting

6.1 Data Sources

A. Production: Statistics (including secondary remelting) from Metal Statistics 1961-71 [32], p.39. [13]

B. Emission Factors: E.P.A. Table 7.12-1. [14]
Units: per unit of material processed.

6.2 Ecologic Magnitudes

See Table S29.6

TABLE S29.6

Ecologic Data for Secondary Magnesium Smelting

Pot Furnace	Parts. Factor lbs/ton	Parts. Emission Tons
	2.464	3.9

[12] (cont.) efficiency of such controls. We adopt the E.P.A.'s factors, applying 90% control.

[13] Production of 3.5×10^3 Tons as surrogate for input quantity.

[14] Average of controlled and uncontrolled factors, since no information is available on this question. Thus we assume 50% subject to control on particulate emissions.

S68: BRICKS, FIRECLAY AND REFRACTORY GOODS

Emissions Source 1: Fuels

1.1 Data Sources

A. Consumption: Census

B. Emission Factors: G.F.

1.2 Ecologic Magnitudes

PF matrix (Derv and M.S. only)

Emissions Source 2: Brick Manufacture

1.1 Data Sources

A. Production: Brick Development Association[1] (B.D.A.), Clean Air Conference Papers, 1969 [28], and Alkali Inspectorate Reports [4].[2]

B. Storage: B.D.A.[3]

C. Emission Factors: E.P.A. Table 8.3-1.[4] See Table S68.3

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[1] Priv.comm., 29.8.73. But whereas B.D.A.'s figures apply to 1968, Clean Air Conference figures are for 1967. The data from this latter source also cover products in the heavy clay industry, and are presented below (* indicates a B.D.A. figure), adjusted on the assumption that the tonnage of all products other than bricks changes in the same proportion to bricks over the year considered.

Table S68.1 Bricks, etc., production 1968

Commodity	Production 10^3 Tons
Bricks	23,300.00
Sanitary Pipes, etc.	944.43
Roofing Tiles, etc.	258.89
Fireclay, etc., Refractories	892.65
Silica Products	57.99
Basic Refractories	222.64
Total	25,676.60

[2] (cont.)

(cont.)

[2] The A.I. [4] in its 104th Report, 1967, p.50, provides a Table showing the numerical distribution of INTERMITTENT kilns by firing methods, for 1967 (etc.). E. Rowden (Clean Air Conference [28] *op.cit.*) remarks that 90% of the building bricks produced in Britain are fired in CONTINUOUS kilns. Note that these kilns (together with CLAMPS) appear to exhaust the range of production techniques. Assume the mass of bricks produced in CLAMPS to be negligible; that numbers of kilns reflect annual output proportions by kiln type, and the following table may be drawn up showing the distribution of production by kiln type and firing method.

Table S68.2

Units: 10^3 Tons

Kiln Type \ Fuel	COAL (66.814%)	OIL (33.186%)	TOTALS
INTERMITTENT	1,557	773	2,330
CONTINUOUS	14,011	6,959	20,970
TOTALS	15,568	7,732	23,300

[3] B.D.A. gives a proportion of 1% of brick production stored annually, or some 233 x 10^3 Tons.

[4] Storage factors apply only to the quantity stored (as opposed to produced). No time-period is, however, specified by E.P.A. and we assume one year appropriate. Controls on emissions <u>other than those from brick storage</u> are assumed 50% over the E.P.A.'s uncontrolled factors. Higher efficiencies are not adopted because from the A.I. Reports it is apparent that control has largely been achieved in the past by a switch from intermittent to continuous kilns and to less polluting types of fuels. Thus pollution controls are to a considerable extent implicit in the current distribution of production by furnace type and firing techniques.

S68: BRICKS, FIRECLAY AND REFRACTORY GOODS (cont.)

2.2 Ecologic Magnitudes

See Tables S68.4 and S68.6 and NF matrix

TABLE S68.3

Emission Factors for Brick Production and Storage

Units: lb/Ton

Type of Process	Parts.	SO_x	CO	HCO	NO_x	Fluorides
1. Raw Material Handling						
Dryers, Grinders, etc.	53.76	-	-	-	-	-
Storage	38.08	-	-	-	-	-
2. Curing and Firing						
Tunnel Kilns						
Gas-fired	0.0225	Neg.	0.45	0.011	0.084	0.56
Oil-fired	0.336	6.72^S	Neg.	0.056	0.616	0.56
Coal-fired	5.04^A	5.65^S	1.07	0.336	0.51	0.56
Periodic Kilns						
Gas-fired	0.062	Neg.	0.062	0.023	0.235	0.56
Oil-fired	0.51	9.91^S	Neg.	0.056	0.95	0.56
Coal-fired	8.07^A	9.91^S	1.79	0.505	0.79	0.56

Notes: A = ash content of 9% (by wt.)
S = sulphur content of 3% (for oil) and 1.4% (for coal); both by wt.

Source: Clean Air Conference and National Coal Board.

TABLE S68.4

Ecologic Magnitudes from Brick Manufacture, 1968

Type of Process	Parts.	SO_x	CO	HCO	NO_x	Fluorides
1. Storage	3,961	-	-	-	-	-
2. Intermittent Kilns						
Coal	5,606	6,541	1,244	351	546	389
Oil	174	3,420	-	19	328	389
3. Continuous Kilns						
Coal	31,525	35,309	6,611	2,101	3,159	389
Oil	1,041	20,877	-	174	1,414	389
	42,307	66,147	7,855	2,645	5,447	1,556

Emissions Source 3: Castable Refractories

3.1 Data Sources

A. Production: See Table S68.1

B. Emission Factors: E.P.A. Table 8.5-1. See Table S68.5 .

3.2 Ecologic Magnitudes

See Tables S68.5 and S68.6

TABLE S.68.5

Particulate Emissions Data for Castable Refractories Manufacture

Type of Process	Emission Factor lb/ton	Ecologic Magnitude Tons
Raw Mat. Drier Raw Mat. Processing Electric Arc Melting Coking Oven Molding and Shake-out	0.34 29.12* 6.05* - 0.34	38,036
Totals	35.85	38,036

* Arithmetic average of controlled factors

TABLE S68.6

Combined Ecologic Magnitudes from Brick and Castables Manufacture

Units: Tons

Ecologic Commodity	Parts.	SO_x	CO	HCO	NO_x	Fluorides
Totals	80,343	66,147	7,855	2,645	5,447	1,556

S70: CEMENT

Emission Source 1: Fuels

1.1 Data Sources

A. Consumption: Census

B. Emission Factors: G.F. (controlled, except Derv and M.S.).

1.2 Ecologic Magnitudes

See PF Matrix.

Emission Source 2: Cement Production

2.1 Data Sources

A. Production: Associated Portland Cement Manufacturers,[1] quoting World Cement Directory [29] and Statistical Summary of the Mineral Industry [8].

B. Emission Factors: E.P.A. Table 8.6-1. Epprs. assumed, efficiency 98%.[2] Table S70.1 below.

2.2 Ecologic Magnitudes

See Table S70.2 below and NF Matrix.

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[1] Priv.comm., 26.9.73. This source states that two thirds of installed kiln capacity for cement in the U.K. in 1970 was for the wet process. This proportion is assumed to apply to the total output for 1968; hence one third was produced by the Dry Process. Total production: 17,592 Tons.

[2] See P.A. Ward [30].

TABLE S70.1

Emission Factors for Cement Manufacture

Ecologic Commodity	Dry Process		Wet Process	
	Kilns lb/ton	Dryers, etc. lb/ton	Kilns lb/ton	Dryers, etc. lb/ton
Particulates	8.23	3.23	7.66	1.08
Sulphur Dioxide (Mineral source)	0.34	-	0.34	-
Nitrogen Oxides	0.09	-	0.09	-

TABLE S70.2

Ecologic Magnitudes from Cement Manufacture

Ecologic Commodity	Dry Process Tons	Wet Process Tons	Total Tons
Particulates	30.0	45.8	75.8
Sulphur Dioxide	0.9	1.8	2.7
Nitrogen Oxides	0.2	0.5	0.7

S71: OTHER BUILDING MATERIALS ETC.

Emission Source 1: Fuels

1.1 Data Sources

A. Consumption: Census

B. Emission Factors: G.F.

1.2. Ecologic Magnitudes

See PF Matrix

Emission Source 2: Gypsum Manufacturing

2.1 Data Sources

A. Production: Natural Environment Research Council [8].[1]

B. Emission Factors: E.P.A. Table 8.14-1[2] Control by Fabric Filter. See Table S71.1 below.

2.2 Ecologic Magnitudes

See Table S71.1 and NF Matrix

TABLE S71.1

Particulate Emission Factors and Magnitudes for Gypsum Manufacture

Type of Process	Emission Factor lb/ton	Ecologic Magnitude Tons
Raw Material Dryer	0.22	379.7
Primary Grinder	0.0011	
Calciner	0.11	
Conveying	0.0011	
Totals	0.3322	379.7

[1] 2,560 x 10^3 Tons

[2] 'Per ton of process throughput'. We assume tonnage produced as proxy.

S81: CONSTRUCTION

Emissions Source 1: Fuels

1.1 Data Sources

A.	Consumption:	Census, by means of certain assumptions.[1] Summary Tables R151 and R156.
B.	Emission Factors:	G.F.

1.2 Ecologic Magnitudes

PF matrix.

Emissions Source 2: Concrete, Batching

2.1 Data Sources

A.	Production:	Cement and Concrete Association.[2]
B.	Emission Factors:	EPA Table 8.10-1. See Table S81.1.[3]

2.2 Ecologic Magnitudes

See Table S81.1.

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[1] Only value quantities are specified in Census for S81; special treatment was necessary since this is a large sector. Because there is no really comparable single industrial sector to construction from which a reliable estimate of unit fuel costs could be obtained it was decided to adopt the average of the prices of all manufacturing industry after consultation with D.T.I. Divided into the respective value quantities in R151 these yield the desired physical inputs. No breakdown in R151 is given for 'Coal' and 'Coke, etc.' elements. An average price was calculated from R156 (Summary Tables) and applied to R151. Here also the aggregate of the ecologic coefficients for derv. and M.S. was applied to their aggregate quantity.

[2] 17 x 10^3 MT. (priv.comm.). C. & C.A. also provide a figure for "ready mix" production but I assume no significant emissions from this source.

[3] Average of "uncontrolled" and "good control".

TABLE S81.1

Ecologic Data for Concrete Batching

Source Magnitude (10^3 yds concrete batched)	Part. Emission Factor (lb/yd^3 concr.)	Ecologic Magnitude (Tons)
68	0.11	3.3

Note: Assumed 0.25 MT cement for each yd^3 concrete batched (C.C.A., priv.comm.)

S83: ELECTRICITY

Emissions Source 1: Fuels

1.1 Data Sources

A. Consumption: Census

B. Emission Factors: E.P.A. Tables 1.1-2[1] (Coal), 1.3-1[2] (Oil), 1.4-1[3] (Gas), subject to control.

1.2 Ecologic Magnitudes

See PF matrix (for controlled magnitudes)

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[1] Furnace size: > 100 x 10^6 Btu/hr heat input. See Table 4.12 . E.P.A. control efficiencies for this source were not applied to the factors in the Table just mentioned. F.F. Ross (C.E.G.B.) suggested in correspondence that these were considerably too low for the U.K. in general. His estimates of 99.3% for stations of 300 MW and over, and 98% for smaller stations and as an overall average of ash collection efficiencies were adopted. I assume this control reduces emissions of other ecologic commodies also by some 50%, though the actual figure is unknown to me. With regard to the removal of sulphur from coal it is known (priv.comm. with Warren Spring) that C.E.G.B. stations do not practice sulphur removal except by stack gas scrubbing, and this only at the Battersea station.

[2] Power Plant factors, see our Table 4.12. Same collection efficiencies as used for coal are applied; though these are not known the influence of the Alkali Inspectorate's Standards are almost certain to ensure control efficiencies of this order.

[3] See Table 4.12. E.P.A.'s sulphur content is taken as realistic for the U.K. in the absence of data. Again, control efficiencies are those for coal.

S85: RAILWAYS

Emissions Source 1: Fuels

1.1 Data Sources

A. Consumption: Digest, Tables 45 (Liquid Fuels),[1] 28 (Coal), and 86 (Coke and Man.Fuel).[2]

B. Emission Factors: E.P.A. Table 3.2.2-1. See Table S85.1 for emissions from diesel combustion.

1.2 Ecologic Magnitudes

See NF matrix

TABLE S85.1

Locomotive Emission Factors for Diesel Fuel Combustion

Units: $lb/10^3$ Gal.

Parts.	SO_2	CO	HCO	NO_x	Ald.	Org. Acids
30.02	68.45	156.12	112.89	444.35	6.61	8.41

---

[1] Gas/Diesel is treated as diesel fuel. Fuel and Burning Oil are treated as equivalent to the Census' 'Other Liquid Fuels'.

[2] Fuel consumption figures used are as follows:

(a) Liquid Fuels

	10^3 Gals
Gas/Diesel	245,640
Fuel Oil	23,500
Burning Oil	5,680

(b) Solid Fuels

	10^3 Tons
Coal	200
Coke and Man. Fuel	60

S86: ROAD TRANSPORT

Emissions Source 1: Fuels

1.1 Data Sources

A.	Consumption:	No data available for fuel consumption either by road transport of the nationalised industries or private road haulage.

S87: OTHER TRANSPORT

Emissions Source 1: Aircraft Flights

1.1 Data Sources

A. Flight Schedules: J. Parker [24], pp.79-82. See Table S87.1.[1]

B. Aircraft Classification & Engines: Jane's All the World's Aircraft [23], and E.P.A. Table 3.2.1-1. See Tables S87.2, S87.6 and S87.7.

C. Aircraft LTO 's Business Monitor [22], for Aircraft Movements.[2] See Table S87.3.

D. Emission Factors: E.P.A. Table 3.2.1-3. See also Tables S87.6, S87.6A, S87.7, S87.7A.

1.2 Ecologic Magnitudes

See Tables S87.8A, S87.8B and S87.8C.[3]

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[1] Average daily LTO 's (Landing-Take-Off Cycles) for Heathrow are calculated from this table by averaging arrivals and departures; seasonal LTO 's by multiplying these figures by the appropriate number of days (91 for Winter and Summer, 91.5 for Autumn and Spring), and annual figures by summation over seasons.

[2] It is assumed that the total number of LTO 's can be calculated as half the number of aircraft movements (i.e. landings + take-offs), since otherwise there would result an accumulation of British aircraft abroad and/or foreign aircraft in the U.K. (or, in the case of internal flights, at certain U.K. airports). See Table S87.2. From the classification of aircraft the percentage of the total number of LTO 's performed by each aircraft type at Heathrow can be calculated (See Table S87.4). These being assumed to reflect the national distribution, total LTO 's for the U.K. by aircraft type can be calculated on the basis of a table of correspondence between Business Monitor aircraft categories and E.P.A. aircraft types (See Table S87.5).

[3] It should be borne in mind that the use of LTO 's to determine aircraft emissions produces only a minimum estimate of ecologic magnitudes from this source.

TABLE S87.1

DAILY HEATHROW FLIGHT SCHEDULE: TYPICAL SEASONAL ACTIVITY

Trident	707	Vanguard	DC-9	BAC 1-11	Viscount	727	Caravelle	737	VC10	DC-8	Islander	747	FSP	Comet	BEE 18	TU 134	Ilyushin 18	Ilyushin 62	DC4	DC6	Brequet	TOTAL	AIRPORT ACTIVITY
90	48	41	40	30	27	22	21	16	15	13	8	8	3	1	1	1	1	1	1	1	1	390	Summer Departures
82	50	36	40	29	28	23	20	16	16	15	8	10	3	1	1	1	1	2	1	1	1	385	Summer Arrivals
86	49	38	40	29	27	22	20	16	15	14	8	9	3	1	1	1	1	1	1	1	1	387	Average = LTO's Daily
75	41	25	30	16	18	17	8	14	16	9	0	8	3	8	1	1	0	2	0	1	1	294	Winter Departures
76	37	28	30	17	14	18	8	12	13	9	0	8	3	9	1	1	0	2	1	1	1	288	Winter Arrivals
75	39	26	30	16	16	17	8	13	14	9	0	8	3	8	1	1	0	2	1	1	1	291	Average = LTO's Daily

Source: Reference [24]

TABLE S87.2

SEASONAL AND ANNUAL LTO's AT HEATHROW, 1968

Trident (3) **	707 (4)	Vanguard (4)	DC-9 (2)	BAC 1-11 (2)	Viscount (4)	727 (3)	Caravelle (2)	737 (2)	VC10 (4)	DC-8 (4)	Islander (2)
7365.75	4026	2928	3202.5	2058.75	1967.25	1784.25	1281	1326.75	1326.75	1052.25	366
7365.75	4026	2928	3202.5	2058.75	1967.25	1784.25	1281	1326.75	1326.75	1052.25	566
7826	4459	3458	3640	2639	2457	2002	1820	1456	1365	1274	728
6825	3549	2366	2730	1456	1456	1547	728	1183	1274	819	0
29382.5	16060	11680	12775	8212.5	7847.5	7117.5	5110	5292.5	5292.5	4197.5	1460

747 (4)	FSP *	Comet (4)	BEE 18 (2)	TU 134 (2)	Ilyushin 18 (4)	Ilyushin 62 (4)	DC4 (4)	DC6 (4)	Brequet (4)	Total	
777.75	..	411.75	91.5	91.5	45.75	137.25	91.5	91.5	91.5	30,515.25	Autumn
777.75	..	411.75	91.5	91.5	45.75	137.25	91.5	91.5	91.5	30,515.25	Spring
819	..	91	91	91	91	91	91	91	91	34,671.00	Summer
728	..	728	91	91	0	182	91	91	91	26,026.00	Winter
3102.5	..	1642.5	365	365	182.5	547.5	365	365	365	121,727.5	Total 1968

* The type of this aircraft was not ascertained so data omitted.

** Bracketed nos. indicate no. of engines of the aircraft.

TABLE S87.3

U.K. AIRCRAFT MOVEMENTS 1968

	Total	Commercial Movements				Non-Commercial Movements					
		Air Transport	Local Pleasure	Empty Charter Positioning	Other Flights	Test and Training	Other Flights By Air Transport Operators	Aero Club	Private	Official	Military
U.K. Reporting Airports	1,279,423	560,251	10,310	16,714	8,654	148,745	11,933	271,935	161,838	7,779	81,264

Source: Reference [22]

TABLE S87.4

Proportion of Total LTO's at Heathrow (by Aircraft) & U.K. Air Transport LTO's

Aircraft	Percentage of 1968 LTO's at Heathrow	"Air Transport" & "Other Comm." LTO's for U.K. as a whole
Trident	24.1379	68,660.86
707	13.1934	37,528.96
Vanguard	9.7431	27,714.49
DC-9	10.4948	29,852.72
BAC 1-11	6.7466	19,190.87
Viscount	6.4468	18,338.08
727	5.8471	16,632.22
Caravelle	4.1979	11,941.03
737	4.3478	12,367.43
VC-10	4.3478	12,367.43
DC-8	3.4483	9,808.78
Islander	1.1994	3,411.72
747	2.5487	7,249.84
Comet	1.3493	3,838.12
BEE 18	0.2999	853.07
TU 134	0.2999	853.07
Ilyushin 18	0.1499	426.39
Ilyushin 62	0.4498	1,279.47
DC-4	0.2999	853.07
DC-6	0.2999	853.07
Brequet	0.2999	853.07
Total	100.1481[1]	284,873.76[2]

[1] Deviation from 100 due to rounding error

[2] Deviation from 284,452 (Table S87.3) due to rounding error

TABLE S87.5

Cross-Classification of LTOC's for U.K. 1968

Business Monitor Categories		EPA Categories: General Aviation Piston	Piston Transport	Business Jets	Military Aircraft[2]	"Aircraft Average"[3]
Commercial Movements[1]	Local Pleasure	5,155				
	Empty Charter Positioning		8,357			
Non-Commercial Movements	Aero Club, Private, & Official	110,388		110,388		
	Military				40,632	
	Other Flights	5,966.5				
	Test & Training					74,372.5

[1] Excludes "Air Transport" & "Other Flights" which are dealt with in Table S87.6

[2] Average of 3 engines and made up of each of EPA's military aircraft in equal proportion

[3] Weighted average of all engines (and emission factors)

TABLE S87.6

ENGINE EMISSION FACTORS
(Commercial Aircraft)

Units: lb/engine

Aircraft	Solid Parts.	SO_x	CO	HCO	NO_2
Turbofan:					
Jumbo Jet	1.30	1.82	46.8	12.2	31.4
Medium Range Jet	0.41	1.01	17.0	4.9	10.2
Long Range Jet	1.21	1.56	47.4	41.2	7.9
Air Carrier:					
Turbo Prop	1.10	0.40	6.6	2.9	2.5
Piston Transport	0.56	0.28	304.0	40.7	0.40

TABLE S87.6A

OTHER ENGINE EMISSION FACTORS
(Non-Commercial Aircraft)

Units: lb/engine

Aircraft	Solid Parts	SO_x	CO	HCO	NO_2
General Aviation Piston	0.02	0.014	12.2	0.40	0.047
Piston Transport	0.56	0.28	304.0	40.7	0.40
Military	0.563	0.437	57.6	11.01	1.897
Test Aircraft & Training*	..	..	..	..	..
General Aviation & Bus.	0.065	0.192	14.0	2.00	0.824

* Only emissions per LTO Cycle calculated.

TABLE S87.7

AIRCRAFT EMISSION FACTORS
(Commercial Aircraft)

Units: lb/Aircraft/LTO

Aircraft		Solid Parts	SO_x	CO	HCO	NO_2
Jumbo	747	5.20	7.28	187.20	48.80	125.60
Medium Range Jet	DC-9	0.82	2.02	34.00	9.80	20.40
	BAC 1-11	0.82	2.02	34.00	9.80	20.40
	727	1.23	3.03	51.00	14.70	30.60
	Trident	1.23	3.03	51.00	14.70	30.60
	Caravelle	0.82	2.02	34.00	9.80	20.40
	737	0.82	2.02	34.00	9.80	20.40
	Comet	1.64	4.04	68.00	19.60	40.80
	TU 134	0.82	2.02	34.00	9.80	20.40
Long Range Jet	707	4.84	6.24	189.60	164.80	31.60
	VC-10	4.84	6.24	189.60	164.80	31.60
	DC-8	4.84	6.24	189.60	164.80	31.60
	Ilyushin 62	4.84	6.24	189.60	164.80	31.60
Air Carrier	Vanguard	4.40	1.60	26.40	11.60	10.00
	Viscount	4.40	1.60	26.40	11.60	10.00
	Ilyushin 18	4.40	1.60	26.40	11.60	10.00
Piston Trans-port	Islander	1.12	0.56	608.00	81.40	0.80
	BEE 18	1.12	0.56	608.00	81.40	0.80
	DC-4	2.24	1.12	1216.00	162.80	1.60
	DC-6	2.24	1.12	1216.00	162.80	1.60
	Brequet	2.24	1.12	1216.00	162.80	1.60

TABLE S87.7A

AIRCRAFT EMISSION FACTORS
(Non-Commercial Aircraft)

Units: lb/Aircraft/LTO

Aircraft	Solid Parts	SO_x	CO	HCO	NO_2
General Aviation Piston	0.02	0.014	12.2	0.40	0.047
Piston Transport	2.24	1.120	1216.0	162.80	1.60
Military *	1.689	1.311	172.8	33.03	5.691
Test Aircraft & Training	2.36	2.582	306.312	67.689	20.055
General Aviation & Bus.**	0.13	0.384	28.0	4	1.648
SUBTOTAL	6.439	5.411	1735.312	267.919	29.041

* Average of 3 engines assumed.
** Average of 2 engines assumed.

TABLE S87.8A

ESTIMATED TOTAL U.K. AIRCRAFT EMISSIONS
"AIR TRANSPORT" & "OTHER COMM. FLIGHTS"

Units: Imp. Tons/Aircraft Type

Aircraft		No. LTO Cycles	Solid Parts.	SO_x	CO	HCO	NO_2
Medium Range Jet	DC-9	29,852.7	10.928	26.921	453.121	130.606	271.873
	BAC 1-11	19,190.9	7.025	17.306	291.290	83.960	147.774
	727	16,632.2	9.133	22.498	378.680	109.149	227.208
	Trident	68.660.9	37.702	92.876	1,563.262	450.587	937.957
	Caravelle	11,941.0	4.371	10.768	181.247	52.242	108.748
	737	12,367.4	4.527	11.153	187.719	54.107	112.632
	Comet	3,838.7	2.810	6.922	116.514	33.583	69.908
	TU 134	853.7	0.312	0.769	12.949	3.732	7.769
	SUBTOTAL		76.808	189.213	3,184.782	917.966	1,910.869
Long Range Jet	707	3,572.9	81.089	104.545	3,176.562	5,761.062	529.427
	VC10	12,367.4	26.722	34.452	1,046.812	909.887	174.469
	DC-8	9,808.8	21.194	27.325	830.245	721.647	138.374
	Ilyushin 62	1,279.5	2.765	3.564	108.301	94.135	18.050
	SUBTOTAL		131.770	169.886	5,161.920	7,486.731	860.320
Jumbo Jet	747	7,249.8	16.830	23.562	605.876	157.942	406.507
Air Carrier	Vanguard	27,714.5	54.439	19.796	326.635	143.522	123.725
	Viscount	18.338.1	36.022	13.099	216.135	94.968	81.869
	Ilyushin 18	426.4	0.838	0.305	5.025	2.208	1.904
	SUBTOTAL		91.299	33.200	547.795	240.698	207.498
Piston Trans-port	Islander	3,411.7	1.706	0.853	926.033	123.979	1.218
	BEE 18	853.1	0.427	0.213	231.556	31.001	0.305
	DC-4	853.1	0.853	0.427	463.111	62.002	0.609
	DC-6	853.1	0.853	0.427	463.111	62.002	0.609
	Brequet	853.1	0.853	0.427	463.111	62.002	0.609
	SUBTOTAL		4.692	2.347	2,546.922	340.986	3.350
	T O T A L :	284,873.9	321.399	418.208	12,047.295	9,144.323	3,388.544

TABLE S87.8B

ESTIMATED TOTAL U.K.
EMISSIONS FROM AIRCRAFT IN REMAINING
BUSINESS MONITOR CLASSES

Units: Imp. Tons/Aircraft Type

	Solid Parts	SO_x	CO	HCO	NO_2	LTO's 1968
"Other Non-Comm." & "Local Pleasure"	.099	.070	60.572	1.986	0.233	11,121.5
"Empty Charter"	8.357	4.179	4,536.657	607.375	5.969	8,357
"Military"	30.637	23.781	3,134.469	599.141	103.231	40,632
"Test & Training"	78.357	85.728	10,170.174	2,247.411	665.866	74,372.5
"Aero Club" & "Private" & "Official"	12.813	37.847	2,759.7	394.243	162.438	220,776
TOTAL	130.263	151.605	20,661.572	3,850.156	937.737	355,259

TABLE S87.8C

U.K. EMISSIONS FROM ALL TYPES OF AIRCRAFT
(COMMERCIAL AND NON-COMMERCIAL)

Units: Imp. Tons

	Solid Parts	SO_x	CO	HCO	NO_2	LTO's 1968 [1]
Total, Table S87.8A	321.399	418.208	12,047.295	9,144.323	3,388.544	284,873.9
Total, Table S87.8B	130.263	151.605	20,661.572	3,850.156	937.737	355,259
GRAND TOTAL	451.662	569.813	32,708.867	12,994.479	4,326.281	640,132.9

[1] These do not sum exactly to Business Monitor figure due to rounding

Emissions Source 2: Water Transport

2.1 Data Sources

A. Fuel Consumption: Digest Tables 44 (Gas/diesel, and fuel oil), 28 (coal), 86 (coke & man. fuel).

B. Emission Factors: General Fuels. For derv see Table S87.9.

2.2 Ecologic Magnitudes

See Table S87.10, and PF matrix.

TABLE S87.9

Emission Factors for Derv used in Water Transport

Units: $lb/10^3$ Gal

Parts.	CO	SO_2	SO_3	HCO	NO_x	Ald.
45.300	82.803	53.693	-	206.975	77.992	-

TABLE S87.10

Ecologic Magnitudes for Water Transport

Units: Tons

Parts.	CO	SO_2	SO_3	HCO	NO_x	Ald.
5.851.7	102.7	2,667.9	0.1	106.5	1753.5	0.3

S89: DISTRIBUTIVE TRADES

Emissions Source 1: Fuels

1.1 Data Sources

A. Consumption: Digest Table 44 (Gas/Diesel and Fuel Oil).

Item: Distributive Trades. See our Table 4.14.

No data for solid and gaseous fuel consumption.

B. Emission Factors: G.F. (O.L.F.) and Table 4.10 of that Section for (for derv).

1.2 Ecologic Magnitudes

See NF Matrix.

S90: MISCELLANEOUS SERVICES

Emissions Source 1: Fuels

1.1 Data Sources

A. Consumption: Digest Table 44.
See classification converter, Table 4.14 .

B. Emission Factors: G.F. except derv, for which see Table 4.10 .

1.2 Ecologic Magnitudes

See NF matrix

Emissions Source 2: Automobile Body Incineration

2.1 Data Sources

A. Production: Working Party on Refuse [21], and Birds Commercial Motors.[1]

B. Emission Factors: EPA Table 2.2-1.[2]

2.2 Ecologic Magnitudes

See Table S90.1

---

[1] Priv.comm. 7.8.73. This Company states that roughly 100,000 cars were incinerated by it in 1968, using 'full pollution controls'. The figure for the U.K. as a whole is about one million (See [21]).

[2] 'With afterburner'.

TABLE S90.1

Ecologic Data for
Automobile Body Incineration

Ecologic Commodity	Controlled Factor (lb/car	Ecologic Magnitude (Tons)
Particulates	1.5	67.0
Nitrogen Oxides	0.02	0.9
Aldehydes	0.06	2.7
Organic Acids	0.07	3.1

TABLE S90.2

Total Ecologic Magnitudes from
All Sources for Sector 90

Units: Tons

Ecologic Commodity	Ecologic Magnitude
1. Particulates	8,757.7
2. Carbon Monoxide	7,269.3
3. Sulphur Dioxide	112,952.9
4. Sulphur Trioxide	1,458.9
5. Hydrocarbons	16,210.1
6. Nitrogen Oxides	22,608.0
7. Aldehydes	422.9
8. Organic Acids	3.1

S91:[1] *CONSUMERS' CURRENT EXPENDITURE*

Emissions Source 1: Fuels

1.1 Data Sources

A. Consumption: Digest Tables 44 (for Gas/diesel and Fuel Oil; See our Table 4.14), 43 (for Motor Spirit),[2] 8 (for Coal),[3] 90 (for Solid Smokeless Fuel),[4] and 54 (for Gas).[5]

B. Emission Factors: G.F. (Fuel Oil and Gas/Diesel,[6] Bituminous and Anthracite Coal, and Smokeless Fuel[7]); see Table 2.14 of "Vehicle Emission Factors".

1.2 Ecologic Magnitudes

See Table S92.

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[1] C.S.O. Sector 93.

[2] In Section 4-4.2b (III) it was decided that 5% of the Digest category "Cars and Motor Cycles" from Table 43, item:Motor Spirit, should be assigned to industrial consumption. The remaining 95% is herewith assigned to Sector 92 (my classification). This quantity is 305,966.5 x 10^3 gallons.

[3]

House and Miners' Coal	21,300 x 10^3 Tons
Anthracite and Dry Steam Coal	1,900 x 10^3 Tons
Total Coal	23,200 x 10^3 Tons

[4] 6,337 x 10^3 Tons, exclusive of anthracite and Dry Steam Coal.

[5] 2,672,000 x 10^3 Therms, item: "Gas Sold:Domestic". Natural (N.Sea) gas may be about one third of the total, (see item: "Gas Purchased") according to Gas Board sources.

[6] Category: "Domestic". - fuel oil and gas/diesel oil quantities are summed in this case.

[7] Category: "Domestic".

TABLE S91

Emissions Data for Final Consumption

Ecologic Commodity	Fuel Oil		Motor Spirit		Coal		Smokeless Fuel		Gas		Totals
	Factor lb/10^3 gal	Mag. Tons	Factor lb/10^3 gal	Mag. Tons	Factor lb/10^3 Tons	Mag. Tons	Factor lb/10^3 Tons	Mag. Tons	Factor lb/10^3 Th	Mag. Tons	Mag. Tons
1. Particulates	12.010	1189.8	26.475	3616.3	16800	159750	11200	41185.0	3.805	4538.8	210279.9
2. Carbon Monoxide	6.000	594.4	3907.533	532709.5	56000	532500	100800	370665.0	4.006	4778.6	1441247.5
3. Sulphur Dioxide	443.392	43927.3	19.895	2717.5	55328	526110	40320	148266.0	0.120	143.1	721163.9
4. Sulphur Trioxide	6.245	618.7	0.000	0.0	0	0	896	3294.8	0.000	0.0	3913.5
5. Hydrocarbons	3.600	356.7	524.797	71683.2	12880	122475	2800	10296.3	1.602	1911.0	206722.2
6. Nitrogen Oxides	14.410	1427.6	99.051	1365.9	5040	47925	3360	12355.5	24.033	28667.9	91741.9
7. Aldehydes	2.400	237.8	0.000	0.0	6	57	0	0.0	0.000	0.0	294.8

S92: EXPENDITURE BY PUBLIC AUTHORITIES*

Emissions Source 1: Fuels

1.1 Data Sources

A. Consumption: Digest: see Classification Converter for Derv and O.L.F.,[1] Motor Spirit,[2] Coal,[3] Coke,[4] and Gas.[5]

B. Emission Factors: G.F., subject to 90% control except derv and M.S.[6]

1.2 Ecologic Magnitudes

See Table S92.

---

* C.S.O. Sector 94.

[1] Census item "derv" equivalenced to Digest item "gas/diesel"; "other liquid fuels" to "fuel oil".

[2] Digest Table 43 provides the quantity of M.S. used by "Services [Armed] and Other Government". The item "Public Service Vehicles and Taxis" appears to cover public utilities; it corresponds well with the sum of the inputs to sectors 82-4 (incl.) from the Census Reports. (Total: ~ 22,087 x 10^3 gals, vs 24,080 x 10^3 gals, of Table 43.) Public Authorities' consumption is thus 51,170 x 10^3 gals, of motor spirit.

[3] 3,000 x 10^3 Tons: Digest Table 28, item: "Public Services".

[4]

Coke Oven Coke*	470 x 10^3 Tons
Gas Coke**	940 x 10^3 Tons
Total Coke	1410 x 10^3 Tons

* Digest Table 86, item: "Public Services"
** Digest Table 87, item: " "

[5]

Public Admin.***	60,000 x 10^3 Th.
Public Lighting	4,000 x 10^3 Th.
Total Gas	64,000 x 10^3 Th.

*** Table 54; item as given.

[6] The following categories in the two sets of Tables were assumed correspondent:

Coal, Fuel Oil	: "Domestic"
M.S., Diesel Oil	: "Vehicles"
Coke (etc.)	: "Industry"
Gas	: "Domestic and Commercial Heating"

Collection efficiencies were assumed not different from the Census Industrial Sectors.

TABLE S92: Public Authorities Current Expenditure - Fuel Emissions

Units: Tons

Ecologic Commodity	Primary Coeff.	Units	Economic Commodity	Quantity	Units	Ecologic Mag.	Reduced Mag.
1. Part-iculates	16.800	lb/ton	Coal a	3,000	10^3 tons	22500.	2250.0
	38.08	lb/ton	Coke etc.	1,410	10^3 tons	23970.	2397.0
	0.045300	lb/gal	Derv	255,519	10^3 gals	5167.4	5167.4
	0.026475	lb/gal	M.S.	51,170	10^3 gals	604.8	604.8
	12.01	lb/10^3 gal	O.L.F.	587,970	10^3 gals	3152.5	315.2
	3.805	lb/10^3 th.	Gas	64,000	10^3 th.	108.7	10.9
Total	-	-	-	-	-	-	10745.3
2. Carbon Monoxide	56.0	as above	Coal a	as above	as above	75000.	7500.
	3.92		Coke etc.			2467.5	246.8
	0.082803		Derv			9445.4	9445.4
	3.907533		M.S.			89262.7	89262.7
	6.0		O.L.F.			1574.9	157.5
	4.006		Gas			114.5	11.4
Total	-	-	-	-	-	-	106623.8
3. Sulphur Dioxide	55.328	as above	Coal a	as above	as above	74100.	7410.
	42.56		Coke etc.			26790.	2679
	0.053693		Derv			6124.8	6124.8
	0.019895		M.S.			454.5	454.5
	443.392		O.L.F.			116384.5	11638.5
	0.120		Gas			3.4	0.3
Total	-	-	-	-	-	-	28307.1
4. Sulphur Trioxide	-	as above	Coal a	as above	as above	-	-
	0.56		Coke etc.			352.5	35.3
	-		Derv			-	-
	-		M.S.			-	-
	6.245		O.L.F.			1639.2	163.9
	-		Gas			-	-
Total	-	-	-	-	-	-	199.2
5. Hydro-carbon	12.88	as above	Coal a	as above	as above	17250.	1725.
	0.13		Coke etc.			81.8	8.2
	0.206975		Derv			23609.8	23609.8
	0.524797		M.S.			11988.3	11988.3
	3.6		O.L.F.			944.9	94.5
	1.602		Gas			45.8	4.6
Total	-	-	-	-	-	-	37430.4
6. Nitrogen Oxides	5.04	as above	Coal a	as above	as above	6750.	675
	15.96		Coke etc.			10046.3	1004.6
	0.077922		Derv			8888.6	8888.6
	0.099051		M.S.			2262.7	2262.7
	14.41		O.L.F.			3782.4	378.2
	20.028		Gas			572.2	57.2
Total	-	-	-	-	-	-	13266.3
7. Aldehydes	0.006	as above	Coal a	as above	as above	8.	0.8
	-		Coke etc.			-	-
	-		Derv			-	-
	-		M.S.			-	-
	2.40		O.L.F.			630.	63.
	-		Gas			-	-
Total							63.8

S92: EXPENDITURE BY PUBLIC AUTHORITIES (cont.)

Emissions Source 2: Refuse Incineration by Local Authorities

2.1 Data Sources

A. Production: Government surveys of refuse disposal, storage and collection.[7]

B. Emission Factors: E.P.A. Table 2.1-1.[8] See Table S92.2

2.2 Ecologic Magnitudes

See Table S92.2 and NF.

TABLE S92.2

Emissions Data for Refuse Incinerators of L.A.'s

Ecologic Commodity	Parts.	CO	SO_2	HCO	NO_x
Controlled Factor	0.7	0.8	0.1	0.04	0.1
Ecologic Magnitude	518.4	592.5	74.1	29.6	74.1

[7] Government Working Party on Refuse Disposal [21]. The Survey strictly applies to England only, and for the year 1966-7. Extrapolation by means of a set of *per capita* waste coefficients (total English refuse ÷ total English population, 1966-7) was made to the whole of the U.K., 1968. Table S92.1 below gives a breakdown of refuse disposal by 'mechanical methods', for 1966-7. The process of separation-incineration is one of salvage-before-incineration with respect to certain materials (such as metals, waste paper, cloth). Another Government publication, [25], estimated only about 313,000 tons of material salvaged by L.A.'s in 1966/7. This constitutes a mere 2% or so of total house refuse. Discussions with a Local Authority official in Edinburgh suggested a proportion of about 10%. Both quantities and proportion have probably risen since 1966/7. We adopt a figure of 3% for 1968/9 and this quantity is subtracted from the total in Table S92.1 below.

Table S92.1 Units: 10^3 Tons

Refuse Type	Separation-Incineration	Direct Incineration	Totals
House Refuse	1,186.428	88.080	1,274.508
Trade etc. Refuse, Delivered Direct	35.851	22.473	58.324
	1,222.279	110.553	1,332.832

(cont.)

[7] (cont.)

This yields a net total of 1292.85 x 10^3 tons of refuse incinerated in 1968 in England and Wales. Expressed in *per capita* terms (using 43m as the relevant population) this becomes

$$\sim 0.03 \text{ ton/cap.} = 67.2 \text{ lbs/cap.}$$

a figure which, though intuitively a substantial underestimate, has been adopted as surrogate for the U.K. total, 1968, vis.,

$$1{,}659{,}000 \times 10^3 \text{ tons}$$

using 55.3m as the U.K. population figure.

[8] E.P.A. 'uncontrolled' factors are employed subject to collection efficiency of 98%, the figure being arrived at from discussions with several Local Authority officiels in the U.K.

4-4.5 National Ecologic Magnitudes: Adjustment Procedure

The National Survey's [33] emissions total for the U.K. for Sulphur Dioxide is adopted as the primary yardstick of accuracy in the present study. The same survey provides also annual estimates of emissions of 'smoke', but since this cannot be identified with any of the E.P.A.'s ecologic quantities, comparison of magnitudes in this case is impossible. As regards the remaining 11 pollutants considered here, no published data on national emissions is available. Our expedient in this situation is to hypothesise that after specific proportional sectorial errors in our initial estimates using E.P.A. factors have been eliminated, any error in the calculation of base-period national emissions of SO_2 arises from the estimation of sectorial pollution levels in such a way that each sector is affected in identical proportions. These same adjustment requirements are then assumed to reflect the proportion by which the estimates of all other ecologic commodities must be diminished. Some justification of this procedure is provided by the following statistical argument.

Denote the true output of SO_2 in the base-year by e . Assume this is a known quantity. For simplicity let there be two sectors, indicated by the subscripts $_1$ and $_2$, with respective true but unknown outputs of SO_2 given by e_1 and e_2 . Then we write

$$e = e_1 + e_2 \tag{4.11}$$

Denote initial estimates of these sectorial quantities by $\hat{e}_1^*$ and $\hat{e}_2^*$ so that

$$\hat{e}^* = \hat{e}_1^* + \hat{e}_2^* \tag{4.12}$$

defines the total initial estimate $\hat{e}^*$ in terms of the sectorial estimates. Assume now that the revised estimates of the quantities in 4.11, $\hat{e}$, $\hat{e}_1$ and $\hat{e}_2$ are related to the initial estimates of 4.12 by two known proportion (henceforth termed 'adjustment proportions'), r_1 and r_2 :

$$\hat{e}_1 = r_1\hat{e}_1^* \tag{4.13}$$

and

$$\hat{e}_2 = r_2\hat{e}_2^* \tag{4.14}$$

This defines a proportion r such that

$$\hat{e}_1^*r_1 + \hat{e}_2^*r_2 = \hat{e}^*r \equiv \hat{e} \tag{4.15}$$

If the adjustment proportions r_1 and r_2 are related by equality then it follows that $r_1 = r_2 = r$. Thus the error proportions[1] would be common to both sectors and reflected in the total. In general, however, $r_1 \neq r_2$. Call these heterogeneous factors "specific sectorial adjustment proportions". Suppose the initial estimates have been corrected for specific sectorial error proportions, so that we have arrived at $\hat{e}$. Since e is known, the proportional error in $\hat{e}$ is given by $(1 - c)$ such that

$$c = e/\hat{e} \tag{4.16}$$

Thus $(1 - c)$ may be regarded as an error proportion *common* to all sectors and existing *in addition* to specific sectorial errors. Hence we may write

$$e = c(r_1\hat{e}_1^* + r_2\hat{e}_2^*) \tag{4.17}$$

---

[1] Error proportion = 1 - adjustment proportion

showing the breakdown of the total error proportion for each sector as the product of specific and common components.

Consider now the case of two pollutants whose base-period estimates are given by $\hat{e}$ and $\hat{f}$ ($\hat{f}$ is an estimate of f incorporating 'specific' error adjustments and estimated in the same way as $\hat{e}$). Allow that the absolute errors in these estimates may differ due to differences in the levels of the ecologic commodities in question, but assume that the *ratio* of the true value to the estimate is the same in each case, and equal to the known common adjustment proportion for $\hat{e}$. This implies that proportional errors for all ecologic commodities are the same, and on this assumption[1] from the initial estimate of f , viz. $\hat{f}$, it is possible to obtain a revised estimate, $\hat{\hat{f}}$:

$$\hat{\hat{f}} = c\hat{f} \tag{4.18}$$

4-4.5a Method of Calculation

We now proceed to describe how emissions of ecologic commodities from economic activity were calculated and adjusted from the empirical data matrices employed in this study. The matrix of first estimates of ecologic magnitudes, $(P\hat{;}W)$ was derived as the sum of two other matrices: NF , a matrix of non-fuels emissions, and $\hat{PF}_0$, a matrix of discharges associated with fuel consumption specifically.[2] Both matrices are of dimensions ecologic commodity x economic sector and are partitioned as follows.

---

[1] The reasonableness of this hypothesis follows from the assumed proportionality of ecologic magnitudes to sectoral outputs.

[2] The division is not exact because fuel emissions from consumption sectors, for example, are included in the NF rather than $\hat{PF}$ matrices for convenience of computation. Emission factors for fuels different from those of the 'ordinary' industrial sectors are employed here. For specific sectors see Basic Datasheets, 4-4.4 above.

$$NF \;=\; \begin{array}{cc} & \begin{array}{cc} 1,2,\ldots\ldots\ldots\ldots,90, & 91,\ldots\ldots,95 \end{array} \\ \begin{array}{c} 1 \\ \vdots \\ 7 \\ 8 \\ \vdots \\ 12 \end{array} & \left[\begin{array}{c:c} NF_1 & \\ & NF_3 \\ \hdashline NF_2 & \end{array}\right] \end{array}$$

$$\hat{PF}_0 \;=\; \begin{array}{cc} & \begin{array}{cc} 1,2,\ldots\ldots\ldots\ldots,90, & 91,\ldots\ldots,95 \end{array} \\ \begin{array}{c} 1 \\ \vdots \\ 7 \\ 8 \\ \vdots \\ 12 \end{array} & \left[\begin{array}{c:c} \hat{PF} & \\ & 0 \\ \hdashline 0 & \end{array}\right] \end{array}$$

$(P,\hat{W})$ is thus written as:

$$(P,\hat{W}) \;=\; \begin{array}{cc} & \begin{array}{cc} 1,2,\ldots\ldots\ldots\ldots,90, & 91,\ldots\ldots,95 \end{array} \\ \begin{array}{c} 1 \\ \vdots \\ 7 \\ 8 \\ \vdots \\ 12 \end{array} & \left[\begin{array}{c:c} NF_1 + \hat{PF} & \\ & NF_3 \\ \hdashline NF_2 & \end{array}\right] \end{array}$$

The first estimate of SO_2 emissions is got by summing row 3 of the $\hat{PF}$ matrix over columns 1 to 90 plus the elment in column 92 of the same row of NF .[1] The common reduction proportion c is obtained by dividing the National Survey (N.S.) figure of 5.09m Tonnes x .984205, by this sum.

Denoting first estimates by a single 'hat' and second estimates by a double 'hat', we have

$$\hat{SO}_2 = \sum_{j=1}^{90} \hat{PF}_{3j} + NF_{3,92} \tag{4.19}$$

$$c = (5.01 \times 10^6)/\hat{SO}_2 \tag{4.20}$$

yielding the identity:

$$\hat{\hat{SO}}_2 = \hat{SO}_2 \cdot c = \hat{SO}_2 \cdot (5.01 \times 10^6)/\hat{SO}_2 =$$

$$= 5.01 \times 10^6 \tag{4.21}$$

The remaining elements of $\hat{PF}$ are likewise reduced in common proportion:

$$\hat{\hat{PF}} = c \cdot \hat{PF} \tag{4.22}$$

NF magnitudes are then added in to get $(\hat{\hat{P,W}})$:

$$(\hat{\hat{P,W}}) = \hat{\hat{PF}} + NF \tag{4.23}$$

$$\begin{array}{cc} & 1,2 \ldots\ldots\ldots\ldots 90,91,\ldots\ldots,95 \\ (\hat{\hat{P,W}}) = \begin{array}{c} 1 \\ \vdots \\ 7 \\ 8 \\ \vdots \\ 12 \end{array} & \left[\begin{array}{c|c} \hat{\hat{PF}} + NF_1 & \\ \hline & NF_3 \\ NF_2 & \end{array} \right] \end{array}$$

---

[1] See p.19 of [33]., Vol I; the figures used are "Industry etc." plus "Power Stations" and exclude "Domestic" (our sector 91) since this is only solid fuels emissions in the Survey. Note that the term "Industry etc." includes "public services", i.e. colliery disposals to national and local authorities. (See p.32 of Digest.) Thus it includes our sectors 1-90 and 92.

4-4.5b Pollution from Final Consumption of Commodities

In the previous section we showed how the calculation of consumption sector ecologic magnitudes were calculated. To determine the total ecologic impact of economic *commodities* it is necessary, as we have seen, to estimate in addition to the pollution arising from their production the quantities of waste residuals generated by the final consumption of those same products. Such an allocation involves much more of a value judgement for certain commodities: should we attribute the pollution from burning petrol in car engines to the petroleum industry or the car industry? Should we allocate pollution from the incineration of domestic waste to the variety of product groups producing plastic etc. containers or to the commodities which use them? There is, in the last resort, nothing but an appeal to ethics or aesthetics to decide these questions; answers fall outside the purview of scientific comment.

The actual allocation chosen in this study is presented below. It serves as an example of the present author's value perspective, nothing more.

Table 4.15
Pollution from Final Consumption of Commodities

Emissions from:	are attributed to CSO Commodity No.[1]:	CSO Description
Coal	3	Coal Mining
Coke and Smokeless Fuel	15	Coke Ovens & Man. Fuel
Fuel Oil	16	Min.Oil Refining
Derv and Motor Spirit	48	Motor Vehicles
Domestic Gas	82	Gas

[1] See Table 4.1 above.

The result of the allocation is a 12 x 90 ecologic commodity x economic commodity matrix, W . Expressed as per unit of these commodities consumed by *all* sectors of final demand yields the required final consumption impacts, a matrix SC . Both these matrices are located in the Statistical Appendix to this study, section 4-7 below.

4-5 IMPACT68

IMPACT68 is the computer program written for the purpose of calculating

(i) secondary ecologic coefficients for production and consumption

and (ii) ecologic impact tables.

In this section we present the algebra underlying these calculations and a description of the programs involved.

4-5.1 Algebra of Calculations

Since there was no intention of using the Stone model for projection purposes in this study the operationally simplest[1] transformation matrix was used, viz. a market share assumption. The following ecologic data matrices were employed, dimensions being added for the reader's convenience.

---

[1] The Commodity Technology assumption requires an operational solution to the existence of negative elements that appear in the A_C matrix. A satisfactory method of 'purging' this matrix has been devised by Almon [34] but the program is not published.

$W_{12\times 90}$ = pollutant magnitudes from final consumption of commodities

$P_{12\times 90}$ = direct pollutant magnitudes from industrial sector production

From these and economic data we obtain:

$\Pi_{12\times 90} = P\hat{g}^{-1}$ = direct pollutant coefficients (industry *secondary* coefficients) from production

$U_{12\times 90} = \Pi D$ = direct pollutant coefficients (commodity *secondary* coefficients) from production

$SI_{12\times 90} = \Pi(I-E)^{-1}D$ = direct and indirect pollution from one unit of each economic commodity (industrial *impact* matrix)

$SC_{12\times 90} = W\hat{CI}^{-1}$ = pollution coefficients from consumption of commodities (consumption *impact* matrix)

$S = SC + SI$

$RI = (SI)(\hat{C1}) = (SI)\hat{f}$

= total pollutant magnitudes from production of commodities

$RC = (SC)(\hat{C1}) = W$

$R = RC + RI$

$BI = (RI)1$ = vector of pollutant magnitudes from industrial production of commodities summed over all commodities

$BC = (RC)1$ = vector of pollutant magnitudes from final consumption of commodities summed over all commodities

$B = BI + BC$

IMPACT68 employs data and uses the output from several other computer programs as input. A schematic representation of the flow of calculations is presented in Table 4.16 below. Individual programs referred to there are now described in summary form.

4-5.2 List of Programs Used and Summary of their Functions

CORRIGENDUM: Reads in derv and motor spirit and O.L.F. for each sector and applies a statistical adjustment procedure, producing new estimates.

ECOCOMP: Uses liquid fuel re-estimates from CORRIGENDUM plus the (unadjusted) non-liquid fuel quantities for each sector, together with the set of primary ecologic coefficients (general fuels) to estimate the PF matrix, this last being of dimension 7 x 90[1] (ecologic commodity x economic sector, for 1968).

PEST: Calculates estimates of total ecologic magnitudes - from fuel and non-fuel sources - using as input $\hat{PF}$ (from ECO-COMP) and NF matrices. These are then subjected to the statistical criterion of correspondence with National Survey figures for SO_2 emissions, and adjusted accordingly producing the 12 x 95 $(\hat{P,W})$ matrix (ecologic commodity x economic sector, for 1968).

IMPACT68: Employing the $(\hat{P,W})$ matrix from PEST and the economic input-output data from CSO, this program calculates secondary ecologic coefficients (for production - 12 x 90 - and for consumption - 12 x 90, the 90 here referring to economic <u>commodities</u> consumed by all <u>five</u> sectors of final demand), ecologic impact tables, and tables of total ecologic magnitudes from production and consumption of economic commodities.

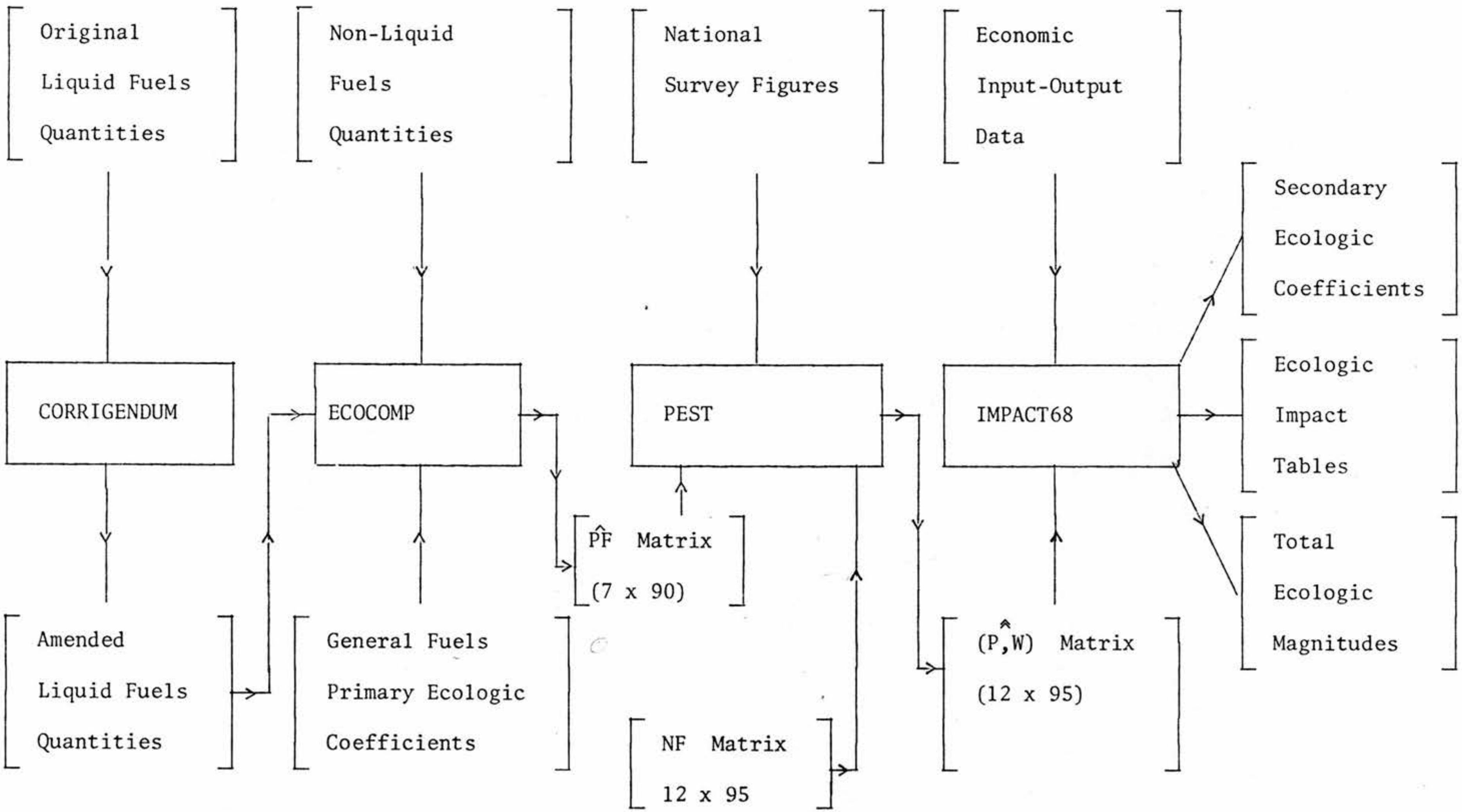
Table 4.16 below presents a schematic representation of the flow of calculations between programs that are just described in summary.

---

[1] Actually 7 x 84, but the remaining six sectors (85-90) are treated, <u>pro tem</u>, <u>as if</u> they had no fuels emissions, then are added in by PEST.

TABLE 4.16

FLOW DIAGRAM FOR IMPACT68 CALCULATIONS



4-6 SUMMARY OF EMPIRICAL FINDINGS

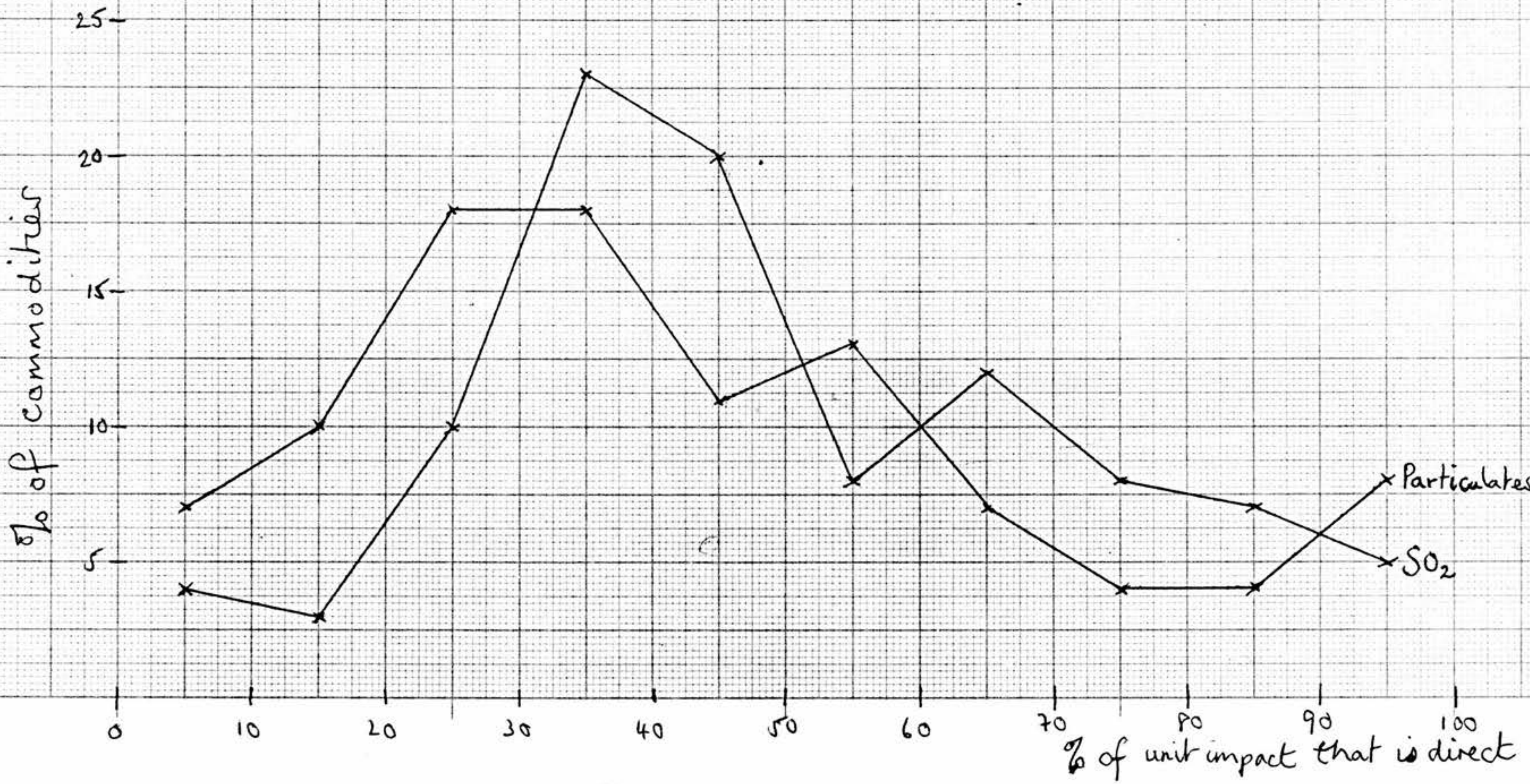
This section provides a numbered set of empirical results, the numbering corresponding to that used in defining the objectives in section 4-1.2 above.

1,2. The matrices of secondary ecologic coefficients for industries and commodities are located as Tables A.3 and A.4 in section 4-7 below. The production (unit) impact matrix (SI) is not presented separately from the total (unit) impact matrix (S), located in Table A.6; but since this differs from SI by only the subtraction of the consumption (unit) impact matrix (SC), presented in Table A.5, the reader can readily obtain one from the other if required. This latter proposition applies, *mutatis mutandis*, to the total impact matrices (i.e. matrices of ecologic magnitudes) RC, RI and R. Tables A.7 and A.8 present RC and R.

3. A graphing of the proportional distribution of economic commodities by *proportions* of total (unit) impact of two pollutants, Particulates and SO_2, is shown on the adjacent page. The modal proportions of both pollutants are in the region of 25%-35% of total impact. SO_2 has a slightly higher proportion of commodities in this modal range (23%) than Particulates (16%). Both distributions have quite a large spread. We can conclude from these results then, that the unit impacts of SO_2 and particulates have a very substantial part (modally, in the region of 60%-70%) of their effect generated *in*directly. A policy of attempting to reduce particulate and SO_2 pollution by

Figure 8

Production Impact: Direct and Indirect Components



tinkering superficially with the technology of a particular commodity's production - say, by requiring the installation of gas cleaning equipment - will *not* therefore have a substantial effect on the overall level of pollution in the economy for a given level of final demand for the commodity in question. It is only by introducing a policy that alters the *input structure* of the commodity in question that this result can be obtained (for example, by altering the requirements of certain process fuels per unit of output - as was the case subsequent to the passing of the Clean Air Act in 1956, bringing a switch from coal- to oil-firing techniques).

4. The rankings of commodities according to production unit and total impacts for seven pollutants are presented in Tables A.10 and A.11. On the adjacent pages we show the 'top ten' and 'bottom ten' commodities for each pollutant according to these rankings.

A few general interpretative remarks are in order here. Firstly, the rather obvious point should be emphasized that the rankings only refer to the *air* pollution impact of industries. This has the consequence that traditional 'heavy' industry does not necessarily figure high in the top ten polluters in terms of unit or total impact. One of the significant effects of the 1956 Clean Air Act was almost certainly to switch the main pollution sink of industry from air to water. Secondly, the air pollution impact of commodities in our sense of the term is substantially effected by non-process fuel inputs: own-vehicle pollution may, for highly dispersed industries or bulky commodities, be a central determinant of pollution generated and may

Table 4.17

'Top Ten' and 'Bottom Ten' commodities for seven pollutants presented in Tables A.10 and A.11

SI Ranking (in order of *decreasing* pollution)	Particulates Economic Commodity	No.	Carbon Monoxide Economic Commodity	No.	Sulphur Dioxide Economic Commodity	No.
1	Paper and Board	74	Stone, etc. Extraction	04	Textile Finishing	63
2	Fertilisers	24	Other Mining etc.	05	Railways	85
3	Coal Mining	63	Agriculture	01	Paper and Board	74
4	Sugar	08	Construction	81	Fertilisers	24
5	Cement	70	Soft Drinks	12	Other Mining etc.	05
6	Dyestuffs etc.	23	Bricks, etc.	68	Sugar	08
7	Man-made Fibres	57	Other Cereal Food	07	Man-Made Fibres	57
8	Bricks etc,	68	Other Building Materials	71	Oils and Fats	10
9	Other Mining etc.	05	Other Food	11	Dyestuffs & Pigments	23
10	Oils and Fats	75	Coke Ovens etc.	15	Stone, etc. Extraction	04
81	Timber Manu.	73	Aluminium, etc.	28	Timber Manu.	73
82	Distributive Trades	89	Road Transport	86	Aluminium, etc.	28
83	Oil Refining	16	Oil Refining	16	Road Transport	86
84	Cutlery & Jewellery	52	Other Non-Ferrous Metals	29	Cutlery & Jewellery	52
85	Water Supply	84	Other Transport	87	Distributive Trades	89
86	Coal Mining	03	Cutlery & Jewellery	52	Other Non-ferrous Metals	29
87	Miscell. Serv.	90	Tobacco	14	Coal Mining	03
88	Electricity	83	Distributive Trades	89	Miscell. Serv.	90
89	Communication	88	Miscell. Serv.	90	Communication	88
90	Other Transport	87	Communication	88	Other Transport	87

(cont.)

Table 4.17 (cont.)

SI Ranking (in order of *decreasing* pollution)	Sulphur Trioxide Economic Commodity	No.	Hydrocarbons Economic Commodity	No.	Nitrogen Oxides Economic Commodity	No.	Aldehydes Economic Commodity	No.
1	Textile Finishing	63	Coke Ovens etc.	15	Railways	85	Textile Finishing	63
2	Oils and Fats	10	Oil Refining	16	Textile Finishing	63	Oils and Fats	10
3	Other Mining etc.	05	Forrestry and Fishing	02	Paper and Board	74	Other Mining	05
4	Stone, etc. Extraction	04	Stone, etc. Extraction	04	Fertilisers	24	Stone, etc. Extraction	04
5	Paper and Board	74	Railways	85	Sugar	08	Paper and Board	74
6	Man-made Fibres	57	Other Mining	05	Other Mining	05	Railways	85
7	Sugar	08	Agriculture	01	Man-made Fibres	57	Man-made Fibres	57
8	Synthetic Resins etc.	22	Gas	82	Cement	70	Sugar	08
9	Other Building Materials	71	Bricks, etc.	68	Dyestuffs and Pigments	23	Synthetic Resins	22
10	Other Paper and Board	76	Fertilisers	24	Stone, etc. Extraction	04	Other Paper	76
81	Coke Ovens etc.	15	Aluminium	28	Tobacco	14	Aluminium	28
82	Other Non-Ferrous Metals	29	Aerospace Equipment	49	Aluminium	28	Bricks, etc.	68
83	Distributive Trades	89	Other Transport	87	Road Transport	86	Electricity	83
84	Bricks, etc.	68	Coal Mining	03	Distributive Trades	89	Distributive Trades	89
85	Miscell. Servs.	90	Other Non-Ferrous Metals	29	Cutlery & Jewellery	52	Other Non-Ferrous Met.	29
86	Electricity	83	Distributive Trades	89	Other Non-Ferrous Metals	29	Water Supply	84
87	Coal Mining	03	Cutlery & Jewellery	52	Coal Mining	03	Miscell. Servs.	90
88	Oil Refining	16	Tobacco	14	Communication	88	Coal Mining	03
89	Communication	88	Miscell. Servs.	90	Miscell. Servs.	98	Communication	88
90	Other Transport	87	Communication	88	Other Transport	87	Other Transport	87

(cont.)

Table 4.17 (cont.)

RI Ranking (in order of *decreasing* pollution)	Particulates Economic Commodity	No.	Carbon Monoxide Economic Commodity	No.	Sulphur Dioxide Economic Commodity	No.
1	Construction	81	Construction	81	Construction	81
2	Other Food	11	Motor Vehicles	48	Coal Mining	03
3	Distributive Trades	89	Coal Mining	03	Distributive Trades	89
4	Motor Vehicles	48	Coke Ovens etc.	15	Other Food	11
5	Coal Mining	03	Other Food	11	Motor Vehicles	48
6	Agriculture	01	Agriculture	01	Miscell. Servs.	90
7	Miscell. Servs.	90	Other Cereal Foodstuffs	07	Railways	85
8	Alcoholic Drink	13	Distributive Trades	89	Alcoholic Drink	13
9	Sugar	08	Miscell. Servs.	90	Coke Ovens etc.	15
10	Hosiery & Knitted Goods	60	Electricity	83	Oil Refining	16
81	Stone, etc. Extraction	04	Wire, etc.	54	Clothing	66
82	Oils and Fats	10	Oils and Fats	10	Wire, etc.	54
83	Packaging Products	75	Engineers' Small Tools	51	Cement	70
84	Iron Castings etc.	26	Fertilisers	24	Aluminium, etc.	28
85	Water Supply	84	Aluminium, etc.	28	Engineers' Small Tools	51
86	Aluminium	28	Iron Castings	26	Packaging Products	75
87	Engineers' Small Tools	51	Cement	70	Iron Castings, etc.	26
88	Bolts, etc.	53	Bolts, etc.	53	Bolts, etc.	53
89	Textile Finishing	63	Cans, etc.	55	Cans, etc.	55
90	Cans etc.	55	Packaging Products	75	Textile Finishing	63

(cont.)

Table 4.17 (cont.)

RI Ranking (in order of *decreasing* pollution)	Sulphur Trioxide Economic Commodity	No.	Hydrocarbons Economic Commodity	No.	Nitrogen Oxides Economic Commodity	No.	Aldehydes Economic Commodity	No.
1	Construction	81	Construction	81	Construction	81	Construction	81
2	Other Food	11	Motor Vehicles	48	Distributive Trades	89	Other Food	11
3	Coke Ovens, etc.	15	Coal Mining	03	Other Food	11	Motor Vehicles	48
4	Motor Vehicles	48	Oil Refining	16	Motor Vehicles	48	Distributive Trades	89
5	Distributive Trades	89	Other Food	11	Railways	85	Miscell. Servs.	90
6	Miscell. Servs.	90	Agriculture	01	Miscell. Servs.	90	Alcoholic Drink	13
7	Alcoholic Drink	13	Distributive Trades	89	Coal Mining	o3	Oil Refining	16
8	Electronics, etc.	43	Miscell. Servs.	90	Gas	82	Other Cereals	07
9	Clothing	66	Coke Ovens, etc.	15	Agriculture	01	Electronics, etc.	43
10	Hosiery, etc.	60	Gas	82	Alcoholic Drink	13	Clothing	66
81	Fertilisers	24	Wire etc. Manufacture	54	Wire, etc. Manufacture	54	Water Supply	84
82	Engineers' Small Tools	51	Oils and Fats	10	Bricks, etc.	68	Engineers' Small Tools	51
83	Aluminium	28	Aluminium	28	Cement	70	Aluminium	28
84	Packaging, etc.	75	Engineers' Small Tools	51	Aluminium	28	Packaging, etc.	75
85	Iron Castings, etc.	26	Iron Castings etc.	26	Engineers' Small Tools	57	Cement	70
86	Cement	70	Cement	70	Packaging, etc.	75	Bricks, etc.	68
87	Bolts, etc.	53	Bolts, etc.	53	Iron Castings, etc.	26	Bolts, etc.	53
88	Bricks, etc.	68	Packaging, etc.	75	Bolts, etc.	53	Iron Castings, etc.	26
89	Cans, etc.	55	Cans, etc.	55	Cans, etc.	55	Textile Finishing	63
90	Textile Finishing	63	Textile Finishing	63	Textile Finishing	63	Cans, etc.	55

completely outweigh the pollution score estimated according to process emissions. Thirdly, some commodities, though themselves not ('directly') highly polluting nonetheless make use of inputs which, relatively speaking, are. This will have the effect of raising their ranking above expectation according to ordinary criteria. (For example, "Oils and Fats" (commodity 75), an extreme case, has a direct pollution impact of 56.4 tons of particulates per unit of output but a total (unit) impact of 374.2 tons per unit of output - over eight times the direct impact!) Fourthly, precisely those industries that, from a process point of view, are traditionally thought of as pollution-intensive, have been, since the advent of the 1956 Act, most stringently monitored and subjected to the strictest controls on our emissions. Fifthly, the statistical dispersion of sample values may mean in some cases that different ranking may reflect very small differences in absolute value of the underlying quantities. Finally, some commodities (e.g. "Road Transport") will show an artificially low pollution impact due simply to the absence of data on them. Here only the indirect pollution effects register in the rankings.

STATISTICAL APPENDIX

TO CHAPTER 4

4-7 STATISTICAL APPENDIX

4-7.1 Computer Listing of Programs

The following programs (in that order) are listed:

1. CORRIGENDUM
2. ECOCOMP
3. PEST
4. IMPACT68
5. MATAB1 [1]

4-7.2 Statistical Tables

The following Tables are included:

---

[1] This is a tabulation subroutine written by the author to suit thesis format.

<u>Note to the listing of Corrigendum</u>

The numbering of sectors in the program does not correspond to CSO numbering because sectors 1 and 2 (Agriculture and Forestry) are omitted. In this program we have the correspondence:

Program Numbering		CSO Numbering
(1, . . . , 82)	$\longleftrightarrow$	(3, . . . 84)

PROGRAM 1

```
C PROGRAM; CORRIGENDUM.
      REAL DCEN(82),LCEN(82),D(82),L(82)
      REAL P(82),RD(82),RL(82),DL(82)
C READ IN DERV+MS(D) AND OLF(L) EXACT QUANTITIES AND
C ESTIMATES FOR EACH SECTOR.
      DO5 I=1,82
5     READ(5,1Ø)DCEN(I),D(I),LCEN(I),L(I)
      CALL MATAB1(DCEN,82,1,4HDCEN,82,1)
      CALL MATAB1(D,82,1,1HD,82,1)
      CALL MATAB1(LCEN,82,1,4HLCEN,82,1)
      CALL MATAB1(L,82,1,1HL,82,1)
C CALCULATE AGGREGATE QUANTITY OF ESTIMATED
C LIQUID FUEL INPUTS TO SECTORS.
      DO15 I=1,82
15    DL(I)=D(I)+L(I)
      CALL MATAB1(DL,82,1,2HDL,82,1)
C CALCULATE TOTAL ESTIMTATED QUANTITY
C OF AGGREGATED FUELS OVER ALL SECTORS.
      T=Ø.Ø
      DO18 I=1,82
18    T=T+DL(I)
      WRITE(6,2Ø)T
C CALCULATE - PROPORTIONS - OF DERV+MS (RD(I))
C AND OLF(RL(I)) IN AGGREGATE EST. OF
C FUEL INPUT TO SECTOR I.
      DO24 I=1,82
      IF(DL(I).NE.Ø.Ø)GOTO22
      RD(I)=Ø.Ø
      RL(I)=Ø.Ø
      GOTO24
22    RD(I)=D(I)/DL(I)
      RL(I)=L(I)/DL(I)
24    CONTINUE
      CALL MATAB1(RD,82,1,2HRD,82,2)
      CALL MATAB1(RL,82,1,2HRL,82,2)
C CALCULATE  PROPORTIONS OF TOTAL ERROR
C QUANTITY ACCRUING TO ITH SECTOR.
      DO 26 I=1,82
26    P(I)=DL(I)/T
      CALL MATAB1(P,82,1,1HP,82,2)
C CALCULATE AGG. ERROR QUAMTITY
C TO BE ADDED TO ITH SECTOR.
      F=8884262.Ø
      C=Ø.Ø
      DO 28 I=1,82
28    C=C+DCEN(I)+LCEN(I)+DL(I)
      E=F-C
      WRITE(6,3Ø)E,F,C
      IF(E.LE.Ø.Ø)GOTO1ØØ
      DO32 I=1,82
32    P(I)=P(I)*E
      CALL MATAB1(P,82,1,3HEIS,82,1)
      GOTO34
1ØØ   WRITE(6,36)
      STOP
C CALCULATE ERROR QUANTITIES OF DERV
C +MS AND OLF TO BE ADDED TO ITH SECTOR.
34    DO4Ø I=1,82
```

CORRIGENDUM (CONT.)

```
            RD(I)=RD(I)*P(I)
   40       RL(I)=RL(I)*P(I)
            CALL MATAB1(RD,82,1,2HMU,82,1)
            CALL MATAB1(RL,82,1,2HOM,82,1)
C ADD DERV AND OLF ERROR QUANTITIES
C TO 'EXACT' CENSUS QUANTITIES + ORIGINAL ESTIMATES
C FOR ITH SECTOR, TO GET FINAL INPUT
C QUANTITIES FOR THAT SECTOR. , DELTA* & LAMDA*.
            DO42 I=1,82
            DCEN(I)=DCEN(I)+RD(I)+D(I)
   42       LCEN(I)=LCEN(I)+RL(I)+L(I)
            CALL MATAB1(DCEN,82,1,4HDEL*,82,1)
            CALL MATAB1(LCEN,82,1,4HLAM*,82,1)
C SUM EACH SECTOR AGGR, FOR CHECK TOTAL F1.
            F1=0.0
            DO50 I=1,82
   50       F1=F1+DCEN(I)+LCEN(I)
            WRITE(6,55)F1,F
C PUNCH DCEN(I),LCEN(I),I=1,82 AS I=3,84.
            DO46 I=1,82
            I1=I+2
   46       WRITE(7,44)I1,DCEN(I),LCEN(I)
   10       FORMAT(4F8.1)
   20       FORMAT(35H0TOTAL EST. QUANTITY OF AGGR. FUELS  ,
           XF12.1,11H 10*3 GALS    )
   30       FORMAT(1H1,'0','0',3H E= ,F12.1/3H F= ,F12.1/
           X3H C= ,F12.1,'  ALL QUANTITIES IN 10**3 GALS')
   36       FORMAT(1H1,' TOTAL ERROR NEGATIVE')
   44       FORMAT(I3,2F10.1)
   55       FORMAT(1H1,' CHECK TOTAL F1: ',F12.1,' = ',F12.1)
            STOP
            END
```

PROGRAM 2

```
C PROGRAM: ECOCOMP.
C PROGRAM FOR CALCULATING BASE-PERIOD ECOLOGIC
C MAGNITUDES FOR 7 ECOLOGIC COMMODITIES
C FROM FUEL CONSUMPTION.  THESE ARE INSERTED
C IN A 7 X 84 MATRIX T(JECO,ISECT).
C     DEFINITIONS OF VARIABLES USED IN PROGRAM AND AS RE
C     F(1) = COALA (1000 TONS)
C     F(2) = COKE
C     F(3) = DERV + M.S. (1000 GALS)
C     F(4) = OTHER LIQD FUELS
C     F(5) = GAS1 (1000 THERMS)
C     F(6) = GAS2
C     F(7) = GAS3
C     F(8)=COALB (1000 TONS)
C     ISECT = INPUT-OUTPUT SECTOR
C     JECO = ECOLOGIC COMMODITY - 1=PARTS,2=CO,3=SO2,4=
C     7=HCHO
      REAL*4    F(8) , T(7,84) ,M(11),B(7)
C     ONLY 8 FUEL QUANTITIES READ IN FOR EACH SECTOR
10    READ(5,15)ISECT,F(1),F(2),(F(I),I=5,8),
     XF(3),F(4)
15    FORMAT(I3,1X,2(F10.1,5X)/4X,4(F10.1,5X)/
     X3X,2F10.1)
      JECO = 0
      IF(ISECT.EQ.95)GO TO 24
  20  JECO = JECO + 1
      IF(JECO.EQ.8)GO TO 10
C     CALCULATE,FIRST IN LBS,ECOLOGIC MAGNITUDES
  35  GO TO(100,200,300,400,500,600,700),JECO
C     PRINT AND PUNCH THE PF MATRIX
C     OF BASE-PERIOD ECOLOGIC MAGNITUDES
C  FROM FUELS BY SECTORS.
C MAGNITUDE OF EACH ECOLOGIC COMMODITY
C CALCULATED FROM ALL FUELS.(SEE TABLE:
C 'EMISSION FACTORS FROM FUELS IN
C COMPUTABLE FORM').
 24   WRITE (6,25)
  25  FORMAT ('1',///)
      CALL ECTAB (T,7,84,6HP MTRX,7)
      DO 26 I=1,7
      DO 26 J=1,84
 26   WRITE(7,30)I,J,T(I,J)
 30   FORMAT(2I3,1F10.1)
      DO 27 I=1,7
      B(I)=0.0
      DO 27 J=1,84
 27   B(I)=T(I,J)+B(I)
      CALL ECTAB(B,7,1,6HBVEC    ,7)
      STOP
C     CALCULATE  PARTICULATES
 100  M(1) = F(1) * 131040.
      M(2) = F(2) * 38080.
C  M(7),M(8),M(9),M(10)        CONTAIN RESPECTIVELY DE
C     AND MTIRE PARTICULATES (AND SO ON FOR OTHER ECOLC
      M(3) = F(4) * 22.820
      M(4) = F(5) * 3.004
      M(5) = F(6) * 3.605
      M(6) = F(7) * 3.805
      M(7) = F(3) * 19.202
      M(8) = F(3)  * 5.634
```

```
      M(9 ) = F(3) * 4.509
      M(10) = F(3) * 5.260
      M(11)=F(8)*28672C.C
C APPLY REDUCTICN FACTCRS TO ALKALI INSP. SECTORS
C (ALL MAGNITUDES EXCEPT DERV AND MS EMISSIONS).
C SECTCR 83 REDUCED  CIFFERENT PERCENTAGE.
 900  IF(ISECT.EQ.15.OR.ISECT.EQ.16.
     XCR.ISECT.EQ.17.OR.ISECT.EQ.25.
     XOR.ISECT.EQ.26.OR.ISECT.EQ.27.
     XOR.ISECT.EQ.28.CR.ISECT.EQ.29.
     XOR.ISECT.EQ.68.OR.ISECT.EQ.69.
     XOR.ISECT.EQ.70.OR.ISECT.EQ.82.
     XOR.ISECT.EC.85)GOTO101
      GOTO104
 101  DO 102 I1=1,6
 102  M(I1)=M(I1)*0.1
      M(11)=M(11)*.1
 104  IF(ISECT.EQ.83)GCTC105
      GOTO800
 105  DO106 I3=1,6
 106  M(I3)=M(I3)*0.02
      DO107 I4=9,11
 107  M(I4)=M(I4)*C.02
 800  DO 50 I = 1,11
  50  M(I) = M(I) / 2240.
      WRITE(6,49)ISECT,JECO,M
  49  FORMAT(2I3,11F10.1)
      WRITE(6,51)
  51  FORMAT(1HO)
C     'T(JECO,ISECT)' IS THE BASE-PERIOD
C ECOLOGIC MAGNITUDE FOR FUELS FOR
C ECOLOGIC COMMODITY JECO AND SECTOR ISECT
C AND IS THE SUM OF THE M(I) FOR
C GIVEN JECO AND ISECT.
      T(JECO,ISECT) = 0.0
      DO 70 I = 1,11
  70  T(JECO,ISECT) = T(JECO,ISECT) + M(I)
      GO TO 20
C     CALCULATE CARBCN MONOXIDE
 200  M(1) = F(1) * 2240.
      M(2) = F(2) * 3920.
      M(3) = F(4) * 4.80
      M(4) = F(5) * 3.405
      M(5) = F(6) * 3.405
      M(6) = F(7) * 4.006
      M(7) = F(3) * 44.714
      M(8) = F(3) * 1797.465
      M( 9) = 0.0
      M(10) = 0.0
      M(11)=F(8)*1120.C
      GO TO 900
C     CALCULATE SULPHUR CIOXIDE
 300  M(1) = F(1) * 59584.
      M(2) = F(2) * 42560.
      M(3) = F(4) * 466.810
      M(4) = F(5) * 0.120
      M(5)=F(6)*0.120
      M(6) = F(7) * 0.120
      M(7) = F(3) * 28.994
      M(8) = F(3) * 9.152
```

```
      M( 9) = 0.0
      M(10) = 0.0
      M(11)=F(8)*65117.0
      GO TO 900
C     CALCULATE SULPHUR TRIOXIDE
  400 M(1) = 0.0
      M(2) = F(2) * 560.0
      M(3) = F(4) * 6.250
      M(4) = 0.0
      M(5) = 0.0
      M(6) = 0.0
      M(7) = 0.0
      M(8) = 0.0
      M(9) = 0.0
      M(10) = 0.0
      M(11)=0.0
      GO TO 900
C     CALCULATE HYDROCARBONS
  500 M(1) = F(1) * 2240.
      M(2) = F(2) * 130.
      M(3) = F(4) * 3.600
      M(4) = F(5) * 0.200
      M(5) = F(6) * 0.601
      M(6) = F(7) * 1.602
      M(7) = F(3) * 111.767
      M(8) = F(3) * 241.407
      M(9) = 0.0
      M(10) = 0.0
      M(11)=F(8)*340.0
      GO TO 900
C     CALCULATE NITROGEN OXIDES
  600 M(1) = F(1) * 16800.
      M(2) = F(2) * 15960.
      M(3) = F(4) * 72.060
      M(4) = F(5) * 120.170
      M(5) = F(6) * 46.069
      M(6) = F(7) * 24.033
      M(7) = F(3) * 42.078
      M(8) = F(3) * 45.563
      M(9) = 0.0
      M(10) = 0.0
      M(11)=F(8)*20160.0
      GO TO 900
C     CALCULATE ALDEHYDES
  700 M(1) = F(1) * 6.
      M(2) = 0.0
      M(3) = F(4) * 1.8
      M(4) = 0.0
      M(5) = 0.0
      M(6) = 0.0
      M(7) = 0.0
      M(8) = 0.0
      M(9) = 0.0
      M(10) = 0.0
      M(11)=F(8)*6.0
      GO TO 900
      END
      SUBROUTINE ECTAB (X,N,M,KF,ND)
      DIMENSION X(ND,M)
      IF (M.GT.1) GO TO 20
```

```
      WRITE (6,1)
 1    FORMAT ('1',////)
      DO 10 I = 1,N,5
      J = MINO (I + 4,N)
      WRITE (6,5) KH, (K,K=I,J)
 5    FORMAT (/A8,5I11)
 10   WRITE (6,15) (X(K,1),K=I,J)
 15   FORMAT (10X,5F11.1)
      RETURN
 20   DO 45 K=1,M,5
      WRITE (6,25)
 25   FORMAT (1H0,1H0)
      L= MINO (K+4,M)
      IF (L.EQ. 20.OR.L.EQ.35.OR.L.EQ.50.
     XOR.L.EQ.65.OR.L.EQ.80)GOTO36
 30   WRITE (6,5) KH,(J,J=K,L)
      GO TO 39
 36   WRITE (6,37)
 37   FORMAT (1H1,///)
      GO TO 30
 39   DO 45 I=1,N
 45   WRITE (6,50) I, (X(I,J),J=K,L)
 50   FORMAT (/I7,4X,5F11.1)
      RETURN
      END
```

PROGRAM 3

```
C PROGRAM: PEST.68
C SET UP NF AND PWF MATRICES FROM PUNCHED
C DATA AND CALCULATE AETA AND EETA.
      REAL  NF(12,95),PWF(12,95),X(95),B(87),B1(87),PF(12,95),
     >TRANM(90,90)
C SET UP NF,PWF AS 12 X 95 NULL MATRICES.
      DO 1 I=1,12
      DO1 J=1,95
1     NF(I,J)=0.0
      DO 2 I=1,12
      DO2 J=1,95
2     PF(I,J)=0.0
C CALC 95-XVECTOR
      READ(10,5001)((TRANM(I,J),J=1,90),I=1,90)
      DO500 J=1,90
      X(J)=0.0
      DO500 I=1,90
500   X(J)=X(J)+TRANM(I,J)
      READ(1,5002)(X(J),J=91,95)
5001  FORMAT(6(13F6.1/),12F6.1)
5002  FORMAT(5F7.1)
C READ IN NON ZERO ELEMENTS OF NF
5     READ(5,10,END=20)J,I,DATUM
10    FORMAT(I8,I3,1F10.1)
      NF(I,J)=DATUM
      GOTO5
 20    NF(I,J)=DATUM
C READ IN NON ZERO ELEMENTS OF PF
21    READ(8,25,END=29)I1,J1,PFEL
25    FORMAT(I3,I3,1F10.1)
      PF(I1,J1)=PFEL
      GOTO21
29    PF(I1,J1)=PFEL
1001  FORMAT(10F10.1)
      CALL MATAB1(NF,12,95,2HNF,12,1)
      CALL MATAB1(PF,12,95,2HPF,12,1)
      SO2=0.0
      DO26 J=1,95
26    SO2=SO2+PF(3,J)
      SO2=SO2+NF(3,92)
      SO2=5010000.0/SO2
      WRITE(6,36)SO2
36    FORMAT('1','SO2 AMENDMENT FACTOR =',F4.1)
C REDUCE ECOLOGIC MAGNITUDES.
      DO260 I=1,12
      DO260 J=1,95
260   PF(I,J)=PF(I,J)*SO2
      DO261 I=1,12
261   NF(I,92)=NF(I,92)*SO2
      DO35 I=1,12
C SUM PF AND NF TO GET P,W MATRIC.
      DO35 J=1,95
35    PWF(I,J)=PF(I,J)+NF(I,J)
      CALL MATAB1(PWF,12,95,3HPWF,12,1)
      WRITE(7,7)((PWF(I,J),I=1,8),J=1,95)
7     FORMAT(8F10.1)
C CALCULATE CONSTRAINT VECTOR B.
C THIS VECTOR IN THE MES SYSTEM IS CALLED ECOLIM
      DO37 I=1,75
37    B(I)=0.0
```

```
      DO38 I=1,12
      I1=I+75
      B(I)=0.0
      DO38 J=1,95
38    B(I1)=B(I1)+PWF(I,J)
      CALL MATAB1(B,87,1,1HB,87,1)
C  PUNCH B IN MPS FORMAT.
      WRITE(6,14)
14    FORMAT('1')
      DO39 J=1,86,2
      J1=J+1
      WRITE(6,60)J,B(J),J1,B(J1)
39    WRITE(7,60)J,B(J),J1,B(J1)
60    FORMAT(4X,'ECOLIM',4X,'COM',I2,5X,F10.1,5X,'COM',I2,5X,F10.1)
C READ IN 95-VECTOR OF ECONOMIC ACTIVITY LEVELS.
      CALL MATAB1(X,95,1,1HX,95,1)
C POSTMULTIPLY (P,W) BY DIAG(X)**-1 TO GET
C (AETA,EETA) MATRIX (HEADED 'A*E*').
      DO45 J1=1,95
      DO 45 I1=1,12
45    PWF(I1,J1)=PWF(I1,J1)/X(J1)
      CALL MATAB1(PWF,12,95,4HA*E*,12,2)
C PUNCH (AETA,EETA) MATRIX IN MPS FORMAT.
      CALL MATAB1(PWF,12,95,3HPWF,12,2)
140    FORMAT(4X,3HACT,I2,5X,3HCOM,
     >I2,5X,F12.6,3X,3HCOM,I2,5X,
     >F12.6,19X)
      DO400 J=1,95
      DO400 I=76,87,2
      I1=I+1
400   WRITE(7,140)J,I,PWF(I,J),
     >I1,PWF(I1,J)
      STOP
      END
```

PROGRAM 4

```
C   PROGRAM: IMPACT68.
      REAL  E(90,90),TRANM(90,90),EETA(12,90),V1(90),V2(90)
      REAL  M(90,90),X(90,90),G(90),Q(90),B(90,90)
      REAL P(12,95),SI(12,90),SC(12,90),W(12,90),F68(90)
      REAL Y(90,90),RI(12,90),RC(12,90),R(12,90),BI(12),BC(12),BT(12)
      REAL C(90,5),U(12,90)
      READ(5,1)((TRANM(I,J),J=1,90),I=1,90)
      READ(5,1)((X(I,J),J=1,90),I=1,90)
      READ(5,65)((C(I,J),J=1,5),I=1,90)
      READ(5,100)((Y(I,J),J=1,90),I=1,90)
      READ(5,4)((P(I,J),J=1,95),I=1,12)
      DO300 K=1,12
      DO300 J=1,90
300   W(K,J)=0.0
      READ(5,30)(W(K,3),K=1,7)
      READ(5,30)(W(K,15),K=1,7)
      READ(5,30)(W(K,16),K=1,7)
      READ(5,30)(W(K,48),K=1,7)
      READ(5,30)(W(K,82),K=1,7)
      DO60 I=1,90
      DO60 J=1,90
60    M(J,I)=TRANM(I,J)
C   CAL. B MATRIX.
      DO5 J=1,90
      G(J)=0.0
      DO5 I=1,90
5     G(J)=G(J)+M(J,I)
      DO101 I=1,90
      DO101 J=1,90
101   X(I,J)=X(I,J)/G(J)
C   CALC. D MATRIX
      DO7 I=1,90
      Q(I)=0.0
      DO7 J=1,90
7     Q(I)=Q(I)+M(J,I)
      DO8 I=1,90
      DO8 J=1,90
8     M(J,I)=M(J,I)/Q(I)
      DO9 I=1,90
      DO9 J=1,90
      E(I,J)=0.0
      DO9 K=1,90
9     E(I,J)=E(I,J)+M(I,K)*X(K,J)
C CALC. INVERSE(I-E).
      DO10 I=1,90
10    E(I,I)=1-E(I,I)
C SET NEGATIVE ELEMENTS IN E TO ZERO
C AND CALCULATE I-E.
      DO801 I=1,90
      DO801 J=1,90
       IF(E(I,J).LT.0.0)E(I,J)=0.0
      IF(I.NE.J)E(I,J)=-E(I,J)
801   CONTINUE
      CALL MINV(E,90,DETE,V1,V2)
C CALC. PYE IMPACT MATRIX.
      DO11 I=1,90
      DO11  J=1,90
      X(I,J)=0.0
      DO11 K=1,90
11    X(I,J)=X(I,J)+E(I,K)*M(K,J)
```

```
      C  PRINT INV(I-E)*D.
            CALL MATAB1(X,90,90,4HIEID,90,2)
      C  DIRECT IND. POLL. COEFFS, PYE.
            DO14 I=1,12
            DO14 J=1,90
      14    P(I,J)=P(I,J)/G(J)
            CALL MATAB1(P,12,90,4HPYEI,12,2)
      C DIRECT COMMOD. POLL. COEFFS, U, FROM PRODUCTION.
            DO50 I=1,12
            DO50 J=1,90
            U(I,J)=0.0
            DO50 K=1,90
      50    U(I,J)=U(I,J)+P(I,K)*M(K,J)
            CALL MATAB1(U,12,90,1HU,12,2)
      C  INDUSTRIAL IMPACT MATRIX SI,DIM=POLL X COMM.
            DO15 I=1,12
            DO15 J=1,90
            SI(I,J)=0.0
            DO15 K=1,90
      15    SI(I,J)=SI(I,J)+P(I,K)*X(K,J)
            CALL MATAB1(SI,12,90,2HSI,12,2)
      C  CALCULATE POLL X CONS COMM IMPACT MATRIX SC
            DO70 I=1,90
            F68(I)=0.0
            DO70 J=1,5
      70    F68(I)=F68(I)+C(I,J)
            DO42 K=1,12
            DO42 N=1,90
      42    SC(K,N)=W(K,N)/F68(N)
            CALL MATAB1(SC,12,90,1HW,12,2)
      C  ADD SI TO SC TO GET S, TOTAL IMPACT MATRIX.
            DO61 I=1,12
            DO61 J=1,90
      61    SI(I,J)=SI(I,J)+SC(I,J)
            CALL MATAB1(SI,12,90,1HS,12,2)
      C MAGNITUDES FROM ECON COMMODITIES.
      C  (A) PRODUCTION
      C  (B) CONSUMPTION
      C  (C) TOTALS
      C  PRODUCTION MAGS.FROM COMMODS.
            DO41 K=1,12
            DO41 I=1,90
      41    RI(K,I)=SI(K,I)*F68(I)
            CALL MATAB1(RI,12,90,4HRI68,12,1)
      C  CONSUMPTION MAGS. 68.
            DO52 K=1,12
            DO52 I=1,90
      52    RC(K,I)=SC(K,I)*F68(I)
            CALL MATAB1(RC,12,90,4HRC68,12,1)
      C  ADD PROD. AND CONS. MAGS. TO GET R MATRIX.
            DO43 K=1,12
            DO43 I=1,90
      43    R(K,I)=RI(K,I)+RC(K,I)
            CALL MATAB1(R,12,90,3HR68,12,1)
      C  TOTAL ECOLOGIC VECTORS (B=R1, ETC.).
            DO44 K=1,12
            BI(K)=0.0
            DO44 I=1,90
      44    BI(K)=BI(K)+RI(K,I)
            CALL MATAB1(BI,12,1,4HBI68,12,1)
```

```
C   CONSUMPTION VECTORS 68.
        DO45 K=1,12
        BC(K)=0.0
        DO45 I=1,90
45      BC(K)=BC(K)+RC(K,I)
        CALL MATAB1(BC,12,1,4HBC68,12,1)
C   TOTAL B VECTORS 68.
        DO47 K=1,12
47      BT(K)=BI(K)+BC(K)
        CALL MATAB1(BT,12,1,4HBT68,12,1)
1       FORMAT(6(13F6.1/),12F6.1)
3       FORMAT(4F7.2)
4       FORMAT(8F10.1)
30      FORMAT(7F8.1)
40      FORMAT(10F8.1)
65      FORMAT(5F7.2)
100     FORMAT(4(20F4.1/),10F4.1)
        STOP
        END
```

PROGRAM 5

```
          SUBROUTINE MATAB1(X,N,M,KH,ND,IFO)
          DIMENSION X(ND,M)
          IF (M.GT.1) GO TO 20
          WRITE (6,1)
    1     FORMAT ('1',////)
          DO 10 I=1,N,5
          J=MINO (I + 4,N)
          GOTO(3,4),IFO
    3     WRITE(6,5)KH,(K,K=I,J)
          GOTO(8,9),IFO
    4     WRITE(6,6) KH,(K,K=I,J)
    7     GOTO(8,9),IFO
    8     WRITE(6,15)(X(K,1),K=I,J)
          GOTO10
    9     WRITE(6,16) (X(K,1),K=I,J)
   10     CONTINUE
          RETURN
   20     DO 45 K=1,M,5
          WRITE (6,25)
   25     FORMAT (1H1,1H0,/)
          L= MINO (K+4,M)
          GOTO(29,30),IFO
   29     WRITE(6,5)KH,(J,J=K,L)
          GOTO39
   30     WRITE(6,6)KH,(J,J=K,L)
   39     DO 45 I=1,N
          IF(I.EQ.26.OR.I.EQ.51.OR.I.EQ.76.OR.I.EQ.101)
         XGOTO(43,44),IFO
          GOTO(48,49),IFO
   43     WRITE(6,61)KH,(J,J=K,L)
   48     WRITE(6,50)I,(X(I,J),J=K,L)
          GOTO45
   44     WRITE(6,62)KH,(J,J=K,L)
   49     WRITE(6,51)I,(X(I,J),J=K,L)
   45     CONTINUE
    5     FORMAT (/A8,5I11)
    6     FORMAT(/A8,5I16)
   15     FORMAT (10X,5F11.1)
   16     FORMAT(10X,5F16.6)
   50     FORMAT(/I7,4X,5F11.1)
   51     FORMAT (/I7,4X,5F16.6)
   61     FORMAT (1H1,/////,A8,5I11)
   62     FORMAT (1H1,/////,A8,5I16)
          RETURN
          END
```

TABLE A.1 : NF MATRIX

NFAA	1	2	3	4	5
1	45804.0	4342.8	610.7	0.0	0.0
2	356206.2	6555.7	0.0	0.0	0.0
3	28653.3	21617.9	0.0	0.0	0.0
4	262.3	234.1	0.0	0.0	0.0
5	70014.1	16072.1	0.0	0.0	0.0
6	16625.7	8699.0	0.0	0.0	0.0
7	76.1	67.4	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

NFAA	6	7	8	9	10
1	0.0	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

NFAA	11	12	13	14	15
1	0.0	0.0	0.0	0.0	53059.0
2	0.0	0.0	0.0	0.0	19248.0
3	0.0	0.0	0.0	0.0	60937.0
4	0.0	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0	60937.0
6	0.0	0.0	0.0	0.0	609.0
7	0.0	0.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	2721.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

NFAA	16	17	18	19	20
1	14251.0	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0	0.0
3	229434.6	40.4	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0	0.0
5	155540.0	0.0	0.0	0.0	0.0
6	49399.0	0.0	0.0	0.0	0.0
7	466.0	0.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0
10	110.0	0.0	0.0	0.0	0.0
11	0.0	3.9	0.0	0.0	0.0
12	0.0	5.5	0.0	0.0	0.0

NFAA	21	22	23	24	25
1	0.0	0.0	0.0	181.7	0.0
2	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	183.5	0.0
7	0.0	0.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	5.0	0.0
10	0.0	0.0	0.0	948.9	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

NFAA	26	27	28	29	30
1	35163.9	30144.3	552.5	3990.5	0.0
2	0.0	65173.3	0.0	0.0	0.0
3	0.0	0.0	0.0	770.2	0.0
4	0.0	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0	0.0
6	0.0	5.5	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0
9	0.0	874.3	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

NFAA	31	32	33	34	35
1	0.0	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

NFAA	36	37	38	39	40
1	0.0	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

NFAA	41	42	43	44	45
1	0.0	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

NFAA	46	47	48	49	50
1	0.0	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

NFAA	51	52	53	54	55
1	0.0	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

NFAA	56	57	58	59	60
1	0.0	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

NEAA	61	62	63	64	65
1	0.0	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

NEAA	66	67	68	69	70
1	0.0	0.0	80343.0	0.0	75.8
2	0.0	0.0	7855.0	0.0	0.0
3	0.0	0.0	66147.0	0.0	2.7
4	0.0	0.0	0.0	0.0	0.0
5	0.0	0.0	2645.0	0.0	0.0
6	0.0	0.0	5119.0	0.0	0.7
7	0.0	0.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	1556.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

NFAA	71	72	73	74	75
1	379.7	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

NFAA	76	77	78	79	80
1	0.0	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

NFAA	81	82	83	84	85
1	3.3	0.0	0.0	0.0	1173421.0
2	0.0	0.0	0.0	0.0	37136.9
3	0.0	0.0	0.0	0.0	540228.1
4	0.0	0.0	0.0	0.0	9.6
5	0.0	0.0	0.0	0.0	32384.6
6	0.0	0.0	0.0	0.0	198864.1
7	0.0	0.0	0.0	0.0	780.8
8	0.0	0.0	0.0	0.0	922.2
9	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

NFAA	86	87	88	89	90
1	0.0	6303.4	0.0	2364.3	8757.7
2	0.0	32811.6	0.0	2602.2	7269.3
3	0.0	3237.7	0.0	23286.6	112952.9
4	0.0	0.1	0.0	291.1	1458.9
5	0.0	13101.0	0.0	5113.3	16210.1
6	0.0	5079.8	0.0	5595.0	22608.0
7	0.0	0.3	0.0	83.8	422.9
8	0.0	0.0	0.0	0.0	3.1
9	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

NFAA	91	92	93	94	95
1	21023.0	10745.3	0.0	0.0	0.0
2	1441247.0	106623.7	0.0	0.0	0.0
3	721124.0	28307.1	0.0	0.0	0.0
4	3914.0	199.2	0.0	0.0	0.0
5	206722.0	37430.4	0.0	0.0	0.0
6	91742.0	13266.3	0.0	0.0	0.0
7	295.0	63.8	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

TABLE A.2 : PWF MATRIX

PWFA	1	2	3	4	5
1	45804.0	4342.8	2187.9	23166.8	21239.4
2	356206.2	6555.7	9522.5	64015.6	11887.3
3	28653.3	21617.9	3210.2	94924.4	36855.5
4	262.3	234.1	39.3	1159.2	378.0
5	70014.1	16072.1	1628.1	12942.0	2679.1
6	16625.7	8699.0	1221.3	18637.2	7288.5
7	76.1	67.4	6.4	329.7	109.7
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

PWFA	6	7	8	9	10
1	10451.6	23060.8	153221.6	[illegible]	10301.8
2	8554.3	118592.0	4775.9	[illegible]	5203.3
3	11173.5	97073.9	144612.3	32366.0	86841.7
4	86.8	1170.9	1038.3	[illegible]	1124.4
5	1817.2	23503.6	3349.4	2165.0	1564.5
6	2790.3	22923.5	31475.2	7728.2	13963.5
7	25.5	335.7	296.4	58.0	324.1
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

PWFA	11	12	13	14	15
1	94691.6	2724.8	121590.7	1654.3	53131.2
2	94635.7	18158.1	23767.9	1895.8	10034.4
3	199212.6	12007.5	214807.7	10097.8	62166.5
4	1994.9	146.0	2187.3	127.6	16.1
5	19237.4	3560.6	7021.2	434.5	61075.5
6	38453.7	2826.6	40909.8	1730.7	328.7
7	573.4	40.8	630.3	36.7	4.9
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	2721.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

PWFA	16	17	18	19	20
1	14587.9	57775.8	23355.0	705.5	2365.5
2	17941.7	24572.6	7025.0	1342.9	8349.2
3	229806.2	113042.8	36095.0	2800.8	17479.1
4	0.0	1185.0	348.1	34.1	223.8
5	158979.6	5955.3	1799.2	277.7	1715.4
6	50252.7	21851.5	7193.4	551.3	3240.9
7	466.0	342.8	101.1	9.4	64.0
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0
10	110.0	0.0	0.0	0.0	0.0
11	0.0	3.9	0.0	0.0	0.0
12	0.0	5.5	0.0	0.0	0.0

PWFA	21	22	23	24	25
1	18978.9	36336.4	97130.3	229505.3	6733.4
2	3403.0	5618.4	5647.2	7071.7	13727.3
3	18524.9	144789.4	68597.2	131172.4	9098.9
4	135.1	1756.1	407.6	374.7	80.5
5	965.3	2251.0	2369.2	4629.5	2751.4
6	4086.1	24541.9	16169.9	34226.3	2374.8
7	39.7	507.2	112.7	113.4	23.6
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	5.0	0.0
10	0.0	0.0	0.0	948.9	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

PWFA	26	27	28	29	30
1	75176.9	133507.7	1599.0	19290.8	1869.6
2	7255.1	88318.7	4350.7	6364.4	3562.1
3	23567.6	155693.4	4975.4	9648.3	2774.6
4	229.4	1964.6	63.6	90.6	27.3
5	1208.9	3774.7	362.0	1237.4	713.0
6	8262.7	52770.0	1153.9	2382.7	847.4
7	11.2	284.1	17.6	21.0	6.7
8	0.0	0.0	0.0	0.0	0.0
9	0.0	874.3	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

PWFA	31	32	33	34	35
1	5649.8	8973.1	3467.8	8268.0	7131.6
2	5580.2	7641.4	2319.4	4125.0	13529.3
3	9430.9	13513.2	13509.8	10721.0	25788.3
4	98.1	139.2	166.9	99.6	313.3
5	1158.8	1577.6	536.7	916.6	2776.1
6	2355.4	3499.2	2603.2	2598.7	5392.3
7	24.7	32.2	45.3	25.5	85.7
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

PWFA	36	37	38	39	40
1	2126.3	29768.4	10166.3	36769.4	8459.2
2	4642.3	30442.9	15834.5	15580.0	11080.0
3	9370.6	43524.5	33349.2	89926.6	18338.5
4	114.5	418.8	402.0	1009.4	199.1
5	962.3	6342.6	3271.6	3841.3	2310.8
6	1813.0	10776.1	7103.2	17431.2	4151.9
7	32.9	112.3	108.2	285.6	56.1
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

PWFA	41	42	43	44	45
1	36892.2	22923.5	14531.3	8204.0	6860.3
2	11520.1	4406.8	17744.9	8610.8	9249.0
3	55149.5	22467.2	70582.0	33117.9	44866.7
4	530.0	165.1	876.6	402.7	572.7
5	2897.7	1229.9	3924.4	1918.2	2091.9
6	11722.6	5256.5	12815.2	6265.5	8254.1
7	149.7	48.3	250.8	115.3	163.9
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

PWFA	46	47	48	49	50
1	12198.7	6731.1	73970.8	48108.8	25216.4
2	7460.6	2165.6	46975.8	10707.6	12980.2
3	59332.5	7453.9	207207.4	59524.5	28752.2
4	746.0	59.9	2389.4	586.1	239.9
5	1829.9	538.2	10910.8	2497.2	2906.0
6	10791.8	1847.8	41217.1	15074.4	6947.1
7	207.7	17.6	678.2	116.4	66.6
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

PWFA	51	52	53	54	55
1	1815.3	1863.6	8016.8	10499.5	2807.9
2	4524.7	1989.7	2201.2	3724.1	2734.7
3	5750.9	10523.8	8248.1	12995.2	4729.8
4	67.7	135.5	63.6	115.3	47.5
5	918.5	445.8	559.2	908.0	598.1
6	1429.8	2082.1	2029.7	3498.4	1323.8
7	18.7	36.7	18.3	31.8	13.5
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

PWFA	56	57	58	59	60
1	28893.7	137446.6	52292.9	97450.4	20284.0
2	39576.2	5612.8	10326.5	9575.0	6504.3
3	96696.3	163985.7	69376.8	86843.9	41891.2
4	1163.3	1419.3	626.2	582.0	448.4
5	8375.4	3323.7	2916.8	3396.3	1685.0
6	21853.4	34351.9	14155.9	19333.3	7961.8
7	318.9	391.1	180.0	171.4	128.4
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

PWFA	61	62	63	64	65
1	13942.5	1231.4	110435.6	18822.9	19122.6
2	4631.4	1154.7	9268.8	8649.4	4389.2
3	26131.9	6163.0	157740.4	34128.5	33111.9
4	286.7	77.1	1471.7	353.0	336.1
5	1085.1	263.9	3859.3	2029.4	1217.9
6	5557.4	1081.7	30821.3	6924.7	6419.0
7	70.7	21.7	428.6	99.6	95.8
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

PWFA	66	67	68	69	70
1	15150.7	6920.2	80671.2	5745.7	69062.9
2	16101.4	4090.9	25326.3	10592.3	4108.5
3	31449.7	10007.3	66508.9	22622.2	41427.5
4	338.7	94.3	0.0	266.5	137.4
5	3400.0	909.5	5994.5	2231.9	1790.2
6	6761.5	2167.9	5950.3	4653.1	10501.7
7	94.7	26.6	0.0	76.7	42.7
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	1556.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

PWFA	71	72	73	74	75
1	80451.7	5333.2	11757.8	573894.7	17213.4
2	49557.3	32009.3	46697.4	19088.2	10859.1
3	130309.7	11568.8	31507.0	438418.9	25359.0
4	1372.5	122.4	353.7	2477.4	239.2
5	10527.9	6235.2	9220.5	12218.6	2405.5
6	29951.4	3590.5	7674.0	102417.6	5394.2
7	327.1	33.7	99.9	702.5	69.2
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

PWFA	76	77	78	79	80
1	19289.2	9945.5	106001.0	6703.8	34791.7
2	7353.6	32987.7	15672.4	10566.1	9919.3
3	58155.4	86588.3	122922.4	35018.5	31480.0
4	676.3	1123.6	1023.7	437.2	217.8
5	1925.3	6864.4	4843.3	2280.1	2442.2
6	10343.8	15720.7	25710.4	6332.9	7648.5
7	194.6	320.4	297.6	125.4	60.6
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

PWFA	81	82	83	84	85
1	65599.0	2207.9	718494.3	25749.0	1173421.0
2	866109.5	36326.3	91902.8	9768.9	37136.9
3	335559.2	3540.0	186299.9	15106.5	540228.1
4	4087.9	37.1	293.1	49.0	9.6
5	168037.0	7291.1	18048.8	2221.0	32384.6
6	94764.9	11044.4	58043.1	4376.5	198864.1
7	1145.3	10.9	97.7	12.4	780.8
8	0.0	0.0	0.0	0.0	922.2
9	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

PWFA	86	87	88	89	90
1	0.0	6303.4	0.0	2364.3	8757.7
2	0.0	32811.6	0.0	2602.2	7269.3
3	0.0	3237.7	0.0	23286.6	112952.9
4	0.0	0.1	0.0	291.1	1458.9
5	0.0	13101.0	0.0	5113.3	16210.1
6	0.0	5079.8	0.0	5595.0	22608.0
7	0.0	0.3	0.0	83.8	422.9
8	0.0	0.0	0.0	0.0	3.1
9	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

PWFA	91	92	93	94	95
1	21028.0	40218.1	0.0	0.0	0.0
2	1441247.0	399077.3	0.0	0.0	0.0
3	721124.0	105949.4	0.0	0.0	0.0
4	3914.0	745.6	0.0	0.0	0.0
5	206722.0	140096.6	0.0	0.0	0.0
6	91742.0	49653.8	0.0	0.0	0.0
7	295.0	238.8	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

TABLE A.3 : Π MATRIX

PYEI	1	2	3	4	5
1	19.152032	36.555527	2.680266	117.538315	452.966064
2	143.940552	55.182648	10.440410	324.787354	252.921371
3	11.980808	181.968857	3.932626	481.605225	784.159912
4	0.109675	1.970538	0.048144	5.881279	8.042558
5	29.274979	135.287018	1.994489	65.662109	57.002151
6	6.951706	73.223907	1.496141	94.557098	155.074554
7	0.031820	0.567340	0.007840	1.672754	2.334044
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

PYEI	6	7	8	9	10
1	31.719604	21.749329	702.529053	123.149124	71.590012
2	25.961472	111.847702	21.897736	25.629379	36.159134
3	33.910507	91.553314	663.054688	101.940536	603.486572
4	0.263430	1.104217	4.760660	0.631813	7.813763
5	5.515030	22.166946	15.352592	6.822073	10.872139
6	8.468297	21.619843	144.315460	24.340866	97.036179
7	0.077390	0.316609	1.359009	0.182678	2.252258
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

	11	12	13	14	15
1	51.398590	21.850845	173.280350	5.339903	230.204559
2	45.940247	145.614304	33.871918	6.119439	86.370895
3	102.704590	96.291122	306.125000	32.594604	269.352295
4	1.082331	1.170810	3.117146	0.411879	0.069757
5	9.899261	28.553329	10.005995	1.402519	264.625244
6	20.872665	22.667206	58.301041	5.586514	3.590555
7	0.311242	0.327185	0.898248	0.118464	0.021231
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	11.789431
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

PYEI	16	17	18	19	20
1	15.310571	59.263489	69.179718	5.801813	13.432716
2	18.830505	25.205322	20.808716	11.043593	47.411713
3	241.190598	115.953522	106.916824	23.032898	99.256714
4	0.0	1.215512	1.031104	0.280426	1.270869
5	166.855286	6.108642	5.329401	2.283718	9.741061
6	52.742172	22.414154	21.307526	4.533719	18.403748
7	0.489085	0.351627	0.299468	0.077303	0.363430
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0
10	0.115449	0.0	0.0	0.0	0.0
11	0.0	0.004000	0.0	0.0	0.0
12	0.0	0.005642	0.0	0.0	0.0

PYEI	21	22	23	24	25
1	115.093399	79.372055	640.663818	1148.675293	20.154007
2	20.636749	12.272649	41.523529	35.393890	41.087723
3	112.340210	316.273193	504.391113	656.518555	27.234283
4	0.819284	3.835968	2.997058	1.875376	0.240948
5	5.853850	4.917011	17.420593	23.170670	8.235323
6	24.779251	53.608521	118.896347	171.302841	7.108110
7	0.240752	1.107911	0.828677	0.567568	0.070638
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.025025	0.0
10	0.0	0.0	0.0	4.749250	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

PYEI	26	27	28	29	30
1	221.697144	71.063965	5.208493	22.797028	22.525314
2	21.395325	47.010590	14.171726	7.521173	42.916901
3	69.501007	82.873047	16.206573	11.401944	33.428940
4	0.676502	1.045724	0.207167	0.107067	0.328916
5	3.565054	2.009211	2.807831	1.462306	8.590370
6	24.366745	28.088608	3.758649	2.815772	10.209648
7	0.033029	0.151222	0.057329	0.024817	0.080723
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.465376	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

PYEI	31	32	33	34	35
1	24.373596	29.940414	27.831467	50.048462	14.982440
2	24.073349	25.496948	13.614777	24.969742	28.423065
3	40.685516	45.089310	108.425400	64.897141	54.177414
4	0.423210	0.464467	1.339487	0.602906	0.658197
5	4.999140	5.263956	4.307385	5.548429	5.832178
6	10.161353	11.675734	20.892456	15.730642	11.328424
7	0.106557	0.107441	0.363564	0.154358	0.180043
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

PYEI	36	37	38	39	40
1	20.805298	37.308594	14.100330	64.507965	17.218094
2	45.423676	38.153931	21.961945	27.333435	22.552551
3	91.688904	54.549042	46.254272	157.766495	37.326706
4	1.120353	0.524880	0.557561	1.770884	0.405254
5	9.415857	7.949148	4.537604	6.739148	4.703469
6	17.739731	13.505633	9.851911	30.581161	8.450894
7	0.321918	0.140745	0.150070	0.501054	0.114188
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0

PYEI	41	42	43	44	45
1	69.898346	64.957870	13.946951	29.363007	18.163406
2	21.826721	12.487453	17.031311	30.818970	24.487762
3	104.489807	63.664856	67.743683	118.532547	118.789642
4	1.004172	0.467841	0.841349	1.441306	1.516287
5	5.490170	3.485143	3.766588	6.865446	5.538541
6	22.210388	14.895241	12.299863	22.424896	21.853653
7	0.283631	0.136867	0.240715	0.412671	0.433944
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

PYEI	46	47	48	49	50
1	23.942535	37.582916	30.129440	57.941493	146.863190
2	14.643006	12.091574	13.133972	12.836069	75.598160
3	116.452621	41.618652	84.308819	71.690399	167.456100
4	1.464133	0.334450	0.973240	0.705890	1.397204
5	3.591567	3.005026	4.444139	3.007589	16.924866
6	21.181183	10.317144	16.788361	18.155365	40.460678
7	0.407655	0.098269	0.276241	0.140190	0.387886
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

PYEI	51	52	53	54	55
1	11.289133	6.874244	67.255020	45.630157	20.013535
2	28.138687	7.339388	13.466446	16.184692	19.848175
3	35.764297	38.819031	69.195465	56.476303	33.712021
4	0.421020	0.499813	0.533557	0.501086	0.338560
5	5.712067	1.644417	4.691277	3.946111	4.263007
6	3.391793	7.680223	17.027679	15.203825	9.435496
7	0.116294	0.135375	0.153523	0.138201	0.096222
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

PYEI	56	57	58	59	60
1	27.345947	423.826416	93.580978	174.330200	45.236942
2	37.456223	17.307465	18.479828	17.128830	14.521780
3	91.516556	505.353516	124.153458	155.356094	93.528137
4	1.100986	4.376514	1.120619	1.041146	1.001117
5	7.926756	10.248873	5.219771	6.075684	3.762005
6	20.682785	105.926590	25.332748	34.585571	17.775848
7	0.301817	1.205985	0.322120	0.306620	0.286671
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

PYEI	61	62	63	64	65
1	69.331192	18.800003	891.328613	70.949219	98.570129
2	23.030319	17.629013	74.808731	32.602371	22.624741
3	129.944809	94.091690	1273.127197	128.641373	170.679947
4	1.425658	1.177100	11.878133	1.330571	1.732473
5	5.395823	4.029010	31.148514	7.649466	6.277837
6	27.634995	16.514511	248.759583	26.101425	33.087631
7	0.351566	0.331298	3.459242	0.375425	0.493815
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

PYEI	66	67	68	69	70
1	19.735199	29.064255	451.433594	20.088516	672.472168
2	20.973572	17.181427	141.725235	31.543594	40.004883
3	40.966171	42.029800	372.181641	67.368347	403.383789
4	0.441188	0.396052	0.0	0.793630	1.337877
5	4.428819	3.819823	33.545044	6.646544	17.431351
6	8.807488	9.104998	33.297684	13.856811	102.256119
7	0.123356	0.111718	0.0	0.228411	0.415774
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	8.707332	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

PYEI	71	72	73	74	75
1	169.016403	14.449232	13.041779	1269.681152	45.646996
2	104.112122	86.722763	71.654938	42.230606	28.796417
3	273.760254	31.343323	48.345993	969.955322	67.247604
4	2.883408	0.331618	0.542736	5.480985	0.634316
5	22.117462	16.393021	14.148418	27.032349	6.378963
6	62.923203	9.727738	11.775388	226.588013	14.304468
7	0.687186	0.091303	0.153292	1.554207	0.183506
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

PYEI	76	77	78	79	80
1	72.244446	8.146714	222.271179	20.041275	103.690140
2	27.541656	27.021393	32.963098	31.587769	30.988129
3	217.811264	70.927521	257.753174	104.689240	93.344299
4	2.532967	0.920381	2.146574	1.307027	0.680413
5	7.210887	5.622874	10.155804	6.816451	7.629494
6	38.740952	12.877387	53.911560	18.932449	23.894104
7	0.728842	0.262451	0.624031	0.374888	0.189316
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

PYEI	81	82	83	84	85
1	0.0	0.0	0.0	0.0	0.0
2	146.664215	62.309296	57.285278	54.453156	70.885512
3	56.822510	6.072045	116.125366	84.205688	1031.166992
4	0.692232	0.063636	0.182696	0.273133	0.018324
5	28.454849	12.506181	11.250265	12.380156	61.814499
6	16.047180	18.944092	36.179703	24.395203	379.584229
7	0.193941	0.018696	0.060899	0.069119	1.490361
8	0.0	0.0	0.0	0.0	1.760261
9	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

PYEI	86	87	88	89	90
1	0.0	0.0	0.0	0.0	0.0
2	0.0	14.317579	0.0	0.447198	0.865599
3	0.0	1.412793	0.0	4.001890	13.449973
4	0.0	0.000044	0.0	0.050027	0.173720
5	0.0	5.716717	0.0	0.878740	1.930233
6	0.0	2.216606	0.0	0.961522	2.692069
7	0.0	0.000131	0.0	0.014401	0.050357
8	0.0	0.0	0.0	0.0	0.000369
9	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

TABLE A.4 : U MATRIX

UAAA	1	2	3	4	5
1	19.235535	36.297012	2.680266	122.184280	414.262207
2	148.940552	56.582016	10.440410	315.902100	233.178726
3	11.980808	179.431732	3.932626	474.083496	717.095459
4	0.109675	1.942763	0.048144	5.744856	7.364525
5	29.274979	133.704742	1.994489	63.942780	52.281494
6	6.951706	72.234756	1.496141	93.262863	142.319695
7	0.031820	0.559347	0.007840	1.629951	2.130013
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.037728	0.010823
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.000002	0.0
12	0.0	0.0	0.0	0.000003	0.0

UAAA	6	7	8	9	10
1	31.357681	24.204910	698.238037	122.408051	65.659302
2	30.838318	106.367187	22.007446	25.987823	44.867599
3	37.697739	90.636749	659.189453	102.337067	526.380371
4	0.316963	1.077161	4.734063	0.641781	6.793758
5	6.465316	21.138855	15.306515	6.884295	12.074668
6	9.301830	21.305969	143.474060	24.371475	85.602585
7	0.092628	0.308996	1.351430	0.185527	1.958089
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.000008	0.0	0.0	0.0
10	0.0	0.001448	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.000011
12	0.0	0.0	0.0	0.0	0.000015

UAAA	11	12	13	14	15
1	52.105545	27.519547	173.262787	5.339903	222.485107
2	46.315323	136.340103	33.369705	6.119439	84.998672
3	105.533066	101.970123	306.091309	32.594604	260.820557
4	1.117131	1.214189	3.116794	0.411879	0.083709
5	9.974175	26.876495	10.005205	1.402519	255.076828
6	21.370377	23.441635	58.294785	5.586514	4.215826
7	0.321134	0.340555	0.898147	0.118464	0.025008
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.000221
10	0.0	0.0	0.0	0.0	11.346977
11	0.000002	0.000036	0.0	0.0	0.000046
12	0.000002	0.000051	0.0	0.0	0.000064

UAAA	16	17	18	19	20
1	16.570251	103.099792	73.018967	18.883820	19.689396
2	19.021912	25.783081	21.229065	13.002887	47.106720
3	237.469772	143.089966	108.846268	36.852921	101.464706
4	0.036675	1.294378	1.034340	0.387605	1.262644
5	162.065948	8.543991	5.459353	2.882993	9.762969
6	51.841843	29.163391	21.873810	7.466168	19.092758
7	0.485115	0.376252	0.299953	0.109297	0.361094
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.002468	0.0	0.0	0.0
10	0.112010	0.178404	0.0	0.0	0.0
11	0.000111	0.003460	0.000040	0.0	0.000034
12	0.000156	0.004879	0.000056	0.0	0.000047

UAAA	21	22	23	24	25
1	111.610840	88.208313	612.132568	1109.136475	27.763000
2	21.099777	13.588853	40.711090	35.978638	38.906982
3	113.282639	310.321777	485.239746	637.789551	41.194641
4	0.844912	3.701030	2.908596	1.835319	0.386265
5	6.390265	5.331428	16.862122	25.892029	8.018473
6	24.817963	53.345322	114.146408	165.391403	9.631721
7	0.248791	1.067366	0.804948	0.555032	0.112281
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.024071	0.000015
10	0.000348	0.005234	0.0	4.720422	0.002838
11	0.000214	0.000014	0.000177	0.000076	0.000112
12	0.000302	0.000020	0.000250	0.000109	0.000153

UAAA	26	27	28	29	30
1	197.550842	71.034286	23.207108	25.252502	23.763184
2	23.789719	46.828384	14.857890	8.363423	39.415085
3	70.687027	82.950256	30.397446	15.893477	39.176163
4	0.712904	1.044056	0.296502	0.148693	0.399671
5	3.566757	2.074125	3.158497	1.665112	7.966473
6	24.109650	27.996490	7.104730	3.838635	10.905648
7	0.053915	0.152294	0.081437	0.036429	0.102082
8	0.0	0.0	0.0	0.0	0.0
9	0.038389	0.459443	0.002260	0.001678	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.000033	0.0
12	0.0	0.0	0.0	0.000047	0.0

UAAA	31	32	33	34	35
1	25.494858	31.379028	31.254608	49.895248	17.497269
2	24.546188	25.884354	18.433929	25.330444	27.956619
3	43.419174	50.917725	97.021637	65.178635	55.485916
4	0.455314	0.534302	1.158019	0.607830	0.661268
5	5.098295	5.371821	4.275087	5.617929	5.777763
6	10.681770	12.621219	19.239716	15.761389	11.710557
7	0.115766	0.129477	0.314704	0.156011	0.181056
8	0.0	0.0	0.0	0.0	0.0
9	0.001027	0.001811	0.0	0.0	0.000098
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

UAAA	36	37	38	39	40
1	21.038788	36.539963	17.153107	56.532928	19.149078
2	43.792282	36.863144	24.666565	27.914169	22.525009
3	90.314362	57.473801	49.578354	134.868408	42.663849
4	1.100727	0.571350	0.585123	1.514721	0.469144
5	9.097539	7.697704	5.065426	6.530200	4.738104
6	17.492218	13.874861	10.758844	26.675461	9.408216
7	0.316041	0.153695	0.157027	0.424466	0.131821
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.000422	0.003616	0.005273	0.0
10	0.0	0.0	0.001371	0.0	0.0
11	0.0	0.000001	0.000003	0.0	0.0
12	0.0	0.000001	0.000005	0.0	0.0

UAAA	41	42	43	44	45
1	64.248474	63.750137	15.138132	29.246918	22.568726
2	22.880920	12.698186	17.843689	30.497940	23.961533
3	102.063156	64.195883	68.179031	116.972275	114.205795
4	1.005175	0.432806	0.840150	1.421033	1.428010
5	5.615686	3.513700	3.937067	6.792836	5.449913
6	21.544373	14.937942	12.514030	22.181641	21.342636
7	0.283963	0.140996	0.240168	0.406597	0.407410
8	0.0	0.0	0.0	0.0	0.0
9	0.000094	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0
UAAA	46	47	48	49	50
1	24.520111	34.951569	30.669800	57.252182	137.187515
2	15.112997	14.831178	19.412552	13.070373	70.780991
3	114.126984	56.747025	34.520554	71.781021	160.330414
4	1.428935	0.560472	0.972415	0.709860	1.365239
5	3.672572	3.560326	4.492563	3.043380	15.724005
6	20.901321	12.617818	16.875412	18.089966	38.578003
7	0.397344	0.161086	0.275659	0.142560	0.376543
8	0.0	0.0	0.0	0.0	0.0
9	0.000183	0.0	0.000564	0.0	0.007550
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

UAAA	51	52	53	54	55
1	14.912098	7.849822	66.705093	46.855408	20.194962
2	27.862823	7.723567	18.952530	16.846436	20.148560
3	41.514618	38.589890	70.708725	56.181488	34.933060
4	0.481863	0.493594	0.559828	0.508630	0.354506
5	5.657521	1.718700	4.739456	3.723129	4.326541
6	10.022503	7.677251	17.300690	15.016494	9.651449
7	0.131761	0.133645	0.159983	0.135109	0.100560
8	0.0	0.0	0.0	0.0	0.0
9	0.003449	0.0	0.002509	0.018848	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

UAAA	56	57	58	59	60
1	28.953323	423.826416	113.028992	173.290207	46.992981
2	36.575027	17.307465	18.644272	17.227966	14.596280
3	89.607803	505.353516	146.705368	155.411469	95.008621
4	1.068310	4.376514	1.313379	1.049145	1.011061
5	7.697990	10.248873	5.563308	6.078410	3.801805
6	20.293137	105.926590	30.094818	34.565491	18.114670
7	0.292000	1.205985	0.374730	0.308277	0.289384
8	0.0	0.0	0.0	0.0	0.0
9	0.003605	0.0	0.0	0.0	0.0
10	0.002264	0.0	0.0	0.0	0.0
11	0.000000	0.0	0.0	0.0	0.0
12	0.000001	0.0	0.0	0.0	0.0

UAAA	61	62	63	64	65
1	69.522644	41.824036	882.057373	72.633713	97.896301
2	23.021271	18.920059	74.151077	32.598358	22.819824
3	129.978012	105.766571	1259.324463	129.602966	169.781052
4	1.424682	1.192720	11.746947	1.334702	1.724234
5	5.397888	4.605333	30.844330	7.664979	6.303347
6	27.642685	19.652740	246.095169	26.404602	32.924011
7	0.351477	0.337924	3.421017	0.375561	0.491456
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.000005	0.0
12	0.0	0.0	0.0	0.000006	0.0

UAAA	66	67	68	69	70
1	19.926422	35.932602	444.218262	20.755432	660.933105
2	20.998383	17.773376	140.012314	31.845871	40.203817
3	41.240799	50.096024	367.826660	67.824600	399.337158
4	0.443765	0.463752	0.039077	0.792124	1.356407
5	4.437155	4.053587	33.252563	6.712999	17.305435
6	8.862326	10.761760	33.423096	13.888591	101.093216
7	0.124099	0.131483	0.009903	0.227934	0.418841
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	8.511267	0.013496	0.000411
10	0.0	0.0	0.006807	0.0	0.078007
11	0.0	0.0	0.0	0.000001	0.000081
12	0.0	0.0	0.0	0.000002	0.000114

UAAA	71	72	73	74	75
1	167.861923	15.346436	18.413040	1227.161865	56.401154
2	104.988785	85.698242	72.661392	41.818497	28.894974
3	267.851807	32.800659	49.539886	940.365479	78.429291
4	2.795861	0.346162	0.555783	5.336836	0.721481
5	22.310242	16.714813	14.352254	26.334427	6.573276
6	61.601227	9.961843	12.050257	219.485870	16.668304
7	0.670692	0.095506	0.156881	1.513537	0.208316
8	0.0	0.0	0.0	0.0	0.0
9	0.003878	0.0	0.0	0.0	0.0
10	0.022552	0.0	0.0	0.0	0.0
11	0.000004	0.0	0.0	0.0	0.0
12	0.000006	0.0	0.0	0.0	0.0

UAAA	76	77	78	79	80
1	83.665146	8.664007	217.365601	29.710999	105.041901
2	27.724792	27.028412	32.904694	31.362015	31.111740
3	212.107391	71.566177	252.927719	110.574219	102.618073
4	2.385322	0.925958	2.111422	1.329404	0.761254
5	7.337016	5.633266	10.076335	6.904060	7.636337
6	38.631332	13.002027	52.916840	20.446899	24.337784
7	0.685799	0.264082	0.613468	0.381014	0.212762
8	0.0	0.0	0.0	0.0	0.0
9	0.000169	0.0	0.000310	0.000271	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.000003	0.000001	0.0
12	0.0	0.0	0.000004	0.000002	0.0

UAAA	81	82	83	84	85
1	0.879045	10.420885	3.181874	1.418410	0.0
2	142.970261	63.007904	56.788254	54.359589	70.835513
3	60.988336	17.503098	117.366150	84.880096	1031.166992
4	0.680722	0.075533	0.199434	0.276987	0.018324
5	27.894302	22.412323	11.470782	12.375906	61.814499
6	17.523865	18.439468	36.319183	24.544327	379.584229
7	0.195078	0.020241	0.066058	0.070301	1.490361
8	0.005375	0.0	0.0	0.0	1.760261
9	0.002199	0.004737	0.001430	0.000028	0.0
10	0.0	0.469263	0.007780	0.005274	0.0
11	0.0	0.000001	0.000035	0.000009	0.0
12	0.0	0.000001	0.000049	0.000013	0.0

UAAA	86	87	88	89	90
1	22.104752	0.0	0.0	5.926616	0.157122
2	16.268311	14.317579	0.0	3.626425	1.043898
3	38.199249	1.412793	0.0	13.838983	14.623042
4	0.378261	0.000044	0.0	0.142147	0.176967
5	3.794306	5.716717	0.0	1.579552	1.989290
6	7.968077	2.216606	0.0	3.109392	3.072869
7	0.106432	0.000131	0.0	0.040656	0.052519
8	0.0	0.0	0.0	0.000005	0.001849
9	0.030185	0.0	0.0	0.001577	-0.000005
10	0.006791	0.0	0.0	0.004595	0.000055
11	0.000030	0.0	0.0	0.000015	-0.000000
12	0.000042	0.0	0.0	0.000021	-0.000000

TABLE A.5 : NONZERO ELEMENTS OF SC MATRIX

	3	15	16	48	82
1	866.773682	618.184570	4.438220	6.045787	12.585629
2	2889.245605	5261.160156	2.217340	406.605469	13.250363
3	2854.574707	2141.064697	163.862625	5.986734	0.396681
4	0.0	47.235474	2.307874	0.0	0.0
5	664.526367	146.163177	1.330581	69.084473	5.299038
6	260.031982	189.505005	5.325274	8.060534	79.460953
7	0.309256	0.0	0.887054	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

TABLE A.6 : S MATRIX

SAAA	1	2	3	4	5
1	148.042297	58.390762	893.617432	155.243759	448.153320
2	237.803833	72.882507	2922.287842	344.057373	252.990128
3	150.387497	240.409668	2916.156494	550.500977	773.936523
4	0.986999	2.346591	0.410430	6.266554	7.767047
5	54.009308	146.323257	673.912342	80.854630	61.889420
6	41.950378	86.951019	277.214600	111.021088	156.093155
7	0.305324	0.707375	0.432931	1.810515	2.253593
8	0.013351	0.015563	0.023909	0.007467	0.007855
9	0.038530	0.013151	0.068068	0.051350	0.053582
10	0.373835	0.024099	0.047271	0.045164	0.075817
11	0.000088	0.000030	0.000023	0.000136	0.000152
12	0.000124	0.000042	0.000032	0.000192	0.000215

SAAA	6	7	8	9	10
1	91.032822	103.637711	797.177734	255.334213	106.402390
2	77.794373	153.815063	82.632690	57.902353	75.283417
3	114.203049	201.712570	773.843750	262.765869	642.395264
4	0.839916	1.943579	5.478032	1.822960	7.995939
5	19.113693	35.754852	32.255661	18.454163	22.771515
6	28.513229	47.273300	172.107224	61.023300	108.506302
7	0.257267	0.572470	1.586129	0.540593	2.317054
8	0.013246	0.011483	0.020861	0.016610	0.006824
9	0.011387	0.015643	0.016102	0.013862	0.009821
10	0.067730	0.066976	0.106401	0.044807	0.040663
11	0.000033	0.000051	0.000043	0.000081	0.000093
12	0.000046	0.000072	0.000061	0.000115	0.000131

SAAA	11	12	13	14	15
1	145.757660	137.050186	224.385239	61.070633	873.501953
2	143.454727	175.913071	62.682465	16.691208	5382.554688
3	227.809158	246.583023	389.423340	102.661713	2551.201172
4	2.039882	2.317247	3.763596	0.919648	47.779160
5	35.891678	41.526413	19.843018	6.061604	419.174072
6	51.883255	56.511017	77.741928	21.792175	240.884639
7	0.601791	0.672593	1.099615	0.272920	0.283735
8	0.012647	0.012761	0.013932	0.009483	0.154257
9	0.030285	0.028145	0.015277	0.006462	0.071605
10	0.154582	0.073593	0.037655	0.010565	11.491277
11	0.000081	0.000291	0.000037	0.000026	0.000133
12	0.000115	0.000411	0.000053	0.000037	0.000187

SAAA	16	17	18	19	20
1	35.808441	156.945786	131.730301	95.700317	113.889786
2	32.261078	51.704056	42.961594	37.445419	71.836594
3	443.462158	260.101318	203.207993	158.020737	247.501968
4	2.558098	2.003362	1.778577	1.368822	2.453707
5	177.220154	30.762924	15.416169	13.879825	23.067154
6	67.179398	56.484009	42.994858	34.525085	51.148026
7	1.466899	0.634728	0.530263	0.406824	0.719609
8	0.005542	0.028660	0.010544	0.013426	0.011967
9	0.008742	0.018537	0.012393	0.022452	0.027120
10	0.147968	0.412246	0.056352	0.061657	0.076983
11	0.000305	0.004165	0.000428	0.000394	0.000499
12	0.000430	0.005874	0.000603	0.000555	0.000704

SAAA	21	22	23	24	25
1	209.514633	163.926392	666.476807	1236.771484	123.715637
2	52.190911	37.437610	61.595398	65.980698	63.193298
3	301.600830	447.408203	581.607666	812.343018	179.064606
4	2.563517	4.761626	3.580230	2.663265	1.407172
5	21.230377	21.985367	28.875839	49.942001	21.691818
6	63.567657	83.348450	136.719086	210.055222	40.718613
7	0.766147	1.405841	1.025004	0.871729	0.430094
8	0.013595	0.013708	0.019834	0.053955	0.016132
9	0.012442	0.012996	0.014731	0.048118	0.018560
10	0.096644	0.113691	0.107929	5.167931	0.099995
11	0.000962	0.000954	0.000982	0.000811	0.000603
12	0.001356	0.001346	0.001384	0.001143	0.000857

SAAA	26	27	28	29	30
1	264.549561	155.049973	60.445511	56.033768	81.109329
2	58.794235	95.543289	31.479874	23.965393	71.070511
3	183.217194	229.376572	89.872803	70.474350	140.164520
4	1.331302	1.918168	0.701285	0.539759	1.216242
5	29.939499	33.189346	11.180531	8.481345	20.102570
6	51.104568	64.639954	21.758362	17.474442	36.661835
7	0.251432	0.410318	0.209146	0.159423	0.327732
8	0.044714	0.050440	0.011007	0.015433	0.024520
9	0.151204	0.755648	0.025262	0.027484	0.119055
10	0.713789	0.709042	0.050639	0.055097	0.145995
11	0.000060	0.000103	0.000056	0.000086	0.000050
12	0.000084	0.000145	0.000079	0.000121	0.000070

SAAA	31	32	33	34	35
1	74.310547	82.212423	89.281845	99.452408	67.825439
2	49.288818	52.324265	48.640976	50.848969	56.078445
3	119.884705	135.387115	195.831161	145.091583	141.525436
4	1.096949	1.216569	1.975275	1.284305	1.396919
5	14.445732	15.563444	15.650822	15.124751	16.040756
6	29.714951	33.678635	43.754440	35.300140	33.148727
7	0.288673	0.318572	0.542704	0.340063	0.378943
8	0.013022	0.017937	0.021188	0.012335	0.015108
9	0.074231	0.088703	0.091855	0.076239	0.102711
10	0.109372	0.127906	0.119748	0.112252	0.119358
11	0.000046	0.000044	0.000046	0.000039	0.000041
12	0.000064	0.000062	0.000065	0.000055	0.000057

SAAA	36	37	38	39	40
1	63.926422	87.056168	64.114136	103.533417	53.647873
2	69.282547	65.057663	66.498230	53.418915	40.666687
3	176.804367	141.025558	132.390808	211.431915	107.952011
4	1.881116	1.273701	1.260387	2.119699	1.027860
5	17.766998	17.813187	17.810684	16.430466	11.311750
6	37.907715	34.576614	31.765137	45.846512	24.650620
7	0.538400	0.344908	0.338368	0.590930	0.295461
8	0.013385	0.013911	0.015826	0.014335	0.009919
9	0.054318	0.091211	0.165657	0.102081	0.031484
10	0.062956	0.107472	0.121396	0.112918	0.040012
11	0.000042	0.000047	0.000048	0.000057	0.000048
12	0.000060	0.000066	0.000068	0.000080	0.000068

SAAA	41	42	43	44	45
1	112.629745	119.943085	53.945786	89.365219	73.235992
2	48.024796	36.388687	39.256714	58.430267	47.453934
3	184.276794	155.923523	145.306107	218.889282	202.920746
4	1.701614	1.238621	1.541233	2.279286	2.177093
5	14.924114	12.432106	11.266359	17.231491	14.349983
6	41.570999	36.968704	29.978180	46.044724	42.152832
7	0.476261	0.359551	0.443758	0.649299	0.624398
8	0.013415	0.014982	0.009287	0.013987	0.012581
9	0.080181	0.052050	0.030519	0.115810	0.047388
10	0.096554	0.066720	0.040076	0.089262	0.063122
11	0.000061	0.000082	0.000061	0.000083	0.000090
12	0.000086	0.000115.	0.000086	0.000117	0.000126

SAAA	46	47	48	49	50
1	63.723465	111.727478	114.142715	108.027466	199.199158
2	40.157745	48.603323	462.858887	35.371964	104.310684
3	191.286377	176.717712	221.014252	156.112808	266.584717
4	2.112527	1.635668	2.147305	1.467116	2.266687
5	12.504877	16.046570	86.968094	10.979566	27.936859
6	39.520142	40.924347	55.541443	38.470291	64.496124
7	0.585373	0.454605	0.599542	0.336948	0.624367
8	0.012767	0.014174	0.015125	0.008146	0.017043
9	0.082198	0.079053	0.097403	0.041701	0.115924
10	0.091760	0.131783	0.132407	0.049955	0.134121
11	0.000044	0.000057	0.000071	0.000032	0.000068
12	0.000062	0.000080	0.000100	0.000046	0.000096

SAAA	51	52	53	54	55
1	50.979477	30.986429	121.325089	114.515381	112.289642
2	48.613083	19.733688	49.280258	54.984924	74.180298
3	102.169876	83.150208	160.698898	164.207809	181.254196
4	0.966333	0.836183	1.260430	1.355550	1.518524
5	13.488203	6.203678	16.251923	18.404465	23.626724
6	25.708237	18.234100	41.047180	43.640305	49.729141
7	0.261195	0.245633	0.346846	0.353211	0.384117
8	0.012767	0.007092	0.019887	0.023267	0.037272
9	0.090912	0.023902	0.144574	0.221577	0.349450
10	0.098805	0.035238	0.146386	0.227103	0.338721
11	0.000032	0.000029	0.000053	0.000074	0.000101
12	0.000046	0.000041	0.000075	0.000104	0.000143

SAAA	56	57	58	59	60
1	86.004730	507.156738	282.915283	362.353516	241.981628
2	67.212097	37.106171	40.470596	42.517181	39.264450
3	183.725555	658.852051	378.080566	396.404785	343.095215
4	1.815643	5.722820	3.277384	3.025160	3.055979
5	19.680557	22.241623	16.197922	17.114456	14.777286
6	44.331421	137.663254	79.147903	85.644272	70.928818
7	0.494657	1.615456	0.940964	0.891138	0.886574
8	0.019013	0.010718	0.008482	0.009433	0.009900
9	0.135879	0.009031	0.007365	0.006972	0.006900
10	0.152834	0.084267	0.035067	0.020596	0.022907
11	0.000082	0.000600	0.000200	0.000084	0.000119
12	0.000115	0.000846	0.000283	0.000119	0.000167

SAAA	61	62	63	64	65
1	228.114975	201.352417	976.031006	157.659302	160.212234
2	48.684616	43.381317	93.400269	52.662094	60.853500
3	340.956543	329.105469	1392.421631	262.873047	261.159180
4	3.164109	3.073826	12.697783	2.434906	2.431824
5	16.265137	15.258098	44.250885	16.421890	18.099335
6	72.953201	67.418427	275.884766	55.883636	54.279770
7	0.863594	0.888991	3.722118	0.704508	0.707636
8	0.012051	0.015174	0.011975	0.016061	0.011643
9	0.007777	0.006932	0.010160	0.014005	0.013986
10	0.022874	0.024665	0.043408	0.032743	0.069711
11	0.000107	0.000130	0.000297	0.000141	0.000150
12	0.000151	0.000184	0.000419	0.000199	0.000211

SAAA	66	67	68	69	70
1	132.913406	116.791946	491.417236	61.977539	727.074707
2	42.681885	41.279892	171.136551	58.267899	72.987717
3	195.090195	168.364822	473.287842	158.344025	535.571777
4	1.708142	1.468077	0.527641	1.395416	2.045175
5	12.805194	12.410158	51.354996	21.023148	36.569839
6	42.295410	36.564301	58.402649	35.657730	136.883057
7	0.497918	0.426257	0.204590	0.435996	0.684734
8	0.011666	0.008475	0.042473	0.022304	0.055227
9	0.007122	0.012441	8.885752	0.066916	0.052049
10	0.018786	0.033346	0.097101	0.056679	0.138486
11	0.000063	0.000102	0.000070	0.000219	0.000170
12	0.000089	0.000144	0.000099	0.000309	0.000239

SAAA	71	72	73	74	75
1	278.925049	72.072174	44.200394	1299.119385	374.202393
2	150.519394	115.033585	94.510162	59.182343	51.759308
3	432.951904	125.349030	98.839691	1036.181152	357.283936
4	3.852365	1.130618	0.965051	5.939179	2.420177
5	43.911087	26.983582	21.924042	37.202774	19.472443
6	101.353989	31.154449	23.954346	242.533493	81.602310
7	1.018703	0.327534	0.279717	1.709795	0.705871
8	0.039098	0.008586	0.007116	0.015942	0.010588
9	0.046228	0.017334	0.018296	0.015808	0.011755
10	0.156123	0.028527	0.022105	0.037821	0.024412
11	0.000120	0.000060	0.000037	0.000135	0.000090
12	0.000169	0.000084	0.000053	0.000190	0.000127

SAAA	76	77	78	79	80
1	332.740479	175.772858	282.254639	104.673645	193.655777
2	48.420731	44.016724	50.600754	52.176773	53.019287
3	445.117676	235.992706	363.376709	262.603271	234.337463
4	3.842660	1.912223	3.027589	2.733424	1.795587
5	18.695343	14.043210	18.687897	17.229797	18.063431
6	92.990494	52.473663	77.165009	51.973938	53.981506
7	1.117756	0.567309	0.890364	0.800636	0.525917
8	0.015628	0.023525	0.012026	0.012488	0.016131
9	0.011036	0.006676	0.015683	0.018709	0.022340
10	0.026292	0.014195	0.045945	0.052582	0.052788
11	0.000118	0.000041	0.000290	0.000306	0.000238
12	0.000166	0.000058	0.000409	0.000431	0.000336

SAAA	81	82	83	84	85
1	61.613983	63.066727	20.532135	28.063751	49.467667
2	205.833435	112.376129	71.895294	70.677109	98.856491
3	167.279663	126.195007	176.621063	131.371506	1111.188721
4	1.550146	0.627321	0.453771	0.560013	0.625383
5	44.999756	58.066589	21.015076	18.203262	72.348099
6	42.269791	124.650955	53.291168	36.263321	399.020020
7	0.442929	0.247742	0.180395	0.150969	1.667058
8	0.017774	0.037147	0.039724	0.007532	1.768355
9	0.213974	0.053566	0.023628	0.059326	0.064235
10	0.058520	0.746800	0.041113	0.054900	0.049587
11	0.000043	0.000063	0.000073	0.000070	0.000037
12	0.000061	0.000089	0.000103	0.000098	0.000051

SAAA	86	87	88	89	90
1	52.302216	7.933516	13.087795	38.733841	22.848068
2	30.112091	21.559494	8.202198	16.227768	10.646889
3	87.534561	20.080566	42.489731	75.587509	53.704071
4	0.740434	0.125522	0.169081	0.537555	0.482601
5	11.301217	10.505504	3.722095	7.015609	5.739833
6	19.320984	6.793715	12.879646	18.634003	12.165393
7	0.222215	0.043441	0.080140	0.175621	0.146475
8	0.005445	0.001616	0.038338	0.022795	0.007922
9	0.045078	0.006908	0.013354	0.012919	0.005487
10	0.039111	0.008003	0.010662	0.022723	0.012848
11	0.000076	0.000012	0.000010	0.000045	0.000021
12	0.000107	0.000017	0.000014	0.000063	0.000030

TABLE A.7 : NONZERO ELEMENTS OF RC68 MATRIX

RC68	3	15	16	48	82
1	161999.9	43582.0	1505.0	9388.5	4549.7
2	539999.9	370911.6	751.9	631417.4	4790.0
3	533519.9	150944.9	55565.8	9296.8	143.4
4	0.0	3330.1	782.6	0.0	0.0
5	124199.9	10304.5	451.2	107281.2	1915.6
6	48600.0	13360.1	1805.8	12517.2	28725.1
7	57.8	0.0	300.8	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

TABLE A.8 : R MATRIX

R68A	1	2	3	4	5
1	146547.0	5273.5	329017.0	4067.4	6094.9
2	235401.9	6588.6	1086175.0	9014.3	3440.7
3	149868.6	21733.0	1078549.0	14423.1	10525.5
4	977.0	212.1	76.7	164.2	105.6
5	53463.8	13227.6	250154.2	2118.4	841.7
6	41526.7	7860.4	100411.4	2908.8	2122.9
7	302.2	63.9	138.7	47.4	30.6
8	13.2	1.4	4.5	0.2	0.1
9	38.1	1.2	12.7	1.6	0.7
10	370.1	2.2	8.8	1.2	1.0
11	0.1	0.0	0.0	0.0	0.0
12	0.1	0.0	0.0	0.0	0.0

R68A	6	7	8	9	10
1	12662.7	58047.4	92153.7	66642.1	2638.8
2	10821.2	86151.8	9552.3	15086.4	1867.0
3	15885.6	112979.1	89456.3	63581.8	15931.4
4	116.8	1088.6	633.3	475.8	198.3
5	2658.7	20026.3	3728.8	4816.5	564.7
6	3966.2	26477.8	19895.6	15927.1	2691.0
7	35.8	320.6	183.4	141.1	57.5
8	1.8	6.4	2.4	4.3	0.2
9	1.6	8.8	1.9	3.6	0.2
10	9.4	37.5	12.3	11.7	1.0
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

R68A	11	12	13	14	15
1	243808.7	13759.8	115715.3	19640.3	105163.8
2	239956.6	17661.7	32325.3	5367.9	750381.6
3	381056.2	24756.9	200825.4	33016.0	330804.5
4	3412.1	232.7	1940.9	295.8	6698.5
5	60036.0	4169.3	10233.0	1949.4	39856.3
6	86785.1	5673.7	40091.5	7008.4	30342.5
7	1006.6	67.5	567.1	87.8	20.0
8	21.2	1.3	7.2	3.0	10.9
9	50.7	2.8	7.9	2.1	5.0
10	258.6	7.4	19.4	3.4	810.1
11	0.1	0.0	0.0	0.0	0.0
12	0.2	0.0	0.0	0.0	0.0

R68A	16	17	18	19	20
1	13647.6	44572.6	32287.1	9062.8	6628.4
2	11691.6	14683.9	10529.9	3546.1	4180.9
3	205943.7	73368.7	49806.3	14964.6	14404.6
4	1650.1	569.0	435.9	129.6	142.8
5	60546.5	8736.7	3778.5	1314.4	1342.5
6	24586.3	16041.4	10538.0	3269.5	2976.8
7	798.2	180.3	130.0	38.5	41.9
8	1.9	8.1	2.6	1.3	0.7
9	3.0	5.3	3.0	2.1	1.6
10	50.2	117.1	13.8	5.8	4.5
11	0.1	1.2	0.1	0.0	0.0
12	0.1	1.7	0.1	0.1	0.0

R68A	21	22	23	24	25
1	23633.2	20113.8	43520.9	22261.9	24743.1
2	5887.1	4599.7	4022.2	1187.7	12639.7
3	34020.6	54397.0	37979.0	14622.2	35812.9
4	289.2	584.3	233.8	47.9	281.4
5	2394.8	2697.6	1885.6	899.0	4338.4
6	7170.4	10226.9	8927.8	3781.0	8143.7
7	86.4	172.5	66.9	15.7	86.0
8	1.5	1.7	1.3	1.0	3.2
9	1.4	1.6	1.0	0.9	3.7
10	10.9	13.9	7.0	93.0	20.0
11	0.1	0.1	0.1	0.0	0.1
12	0.2	0.2	0.1	0.0	0.2

R68A	26	27	28	29	30
1	2486.8	40266.5	1928.2	10472.7	5328.9
2	552.7	24812.6	1004.2	4479.1	4669.3
3	1722.2	59569.1	2866.9	13171.7	9208.8
4	12.5	498.1	22.4	100.9	79.9
5	281.4	8619.3	356.7	1585.2	1320.7
6	480.4	16787.0	694.1	3266.0	2408.7
7	2.4	106.6	6.7	29.8	21.5
8	0.4	13.1	0.4	2.9	1.6
9	1.4	196.2	0.8	5.1	7.8
10	6.7	184.1	1.6	10.3	9.6
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

R68A	31	32	33	34	35
1	14171.0	10391.6	6508.6	13545.4	27150.5
2	9399.4	6613.8	3545.9	6925.6	22448.2
3	22862.0	17112.9	14276.1	19761.5	56652.6
4	209.2	153.8	144.0	174.9	559.2
5	2754.8	1967.2	1140.9	2060.0	6421.1
6	5666.6	4257.0	3189.7	4807.9	13269.4
7	55.0	40.3	39.6	46.3	151.7
8	2.5	2.3	1.5	1.7	6.0
9	14.2	11.2	6.7	10.4	41.1
10	20.9	16.2	8.7	15.3	47.8
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

R68A	36	37	38	39	40
1	4353.4	45356.2	29877.2	20541.0	18031.0
2	4713.1	33895.0	30988.1	10598.3	13668.1
3	12040.4	73474.2	61694.1	41948.1	36282.7
4	128.1	663.6	587.3	420.5	345.5
5	1209.9	9280.7	8299.8	3259.8	3801.9
6	2581.5	18014.4	14802.5	9095.9	8285.1
7	36.7	179.7	157.7	117.2	99.3
8	0.9	7.2	7.4	2.8	3.3
9	3.7	47.5	77.2	20.3	10.6
10	4.3	56.0	56.6	22.4	13.4
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

R68A	41	42	43	44	45
1	38102.6	10746.9	38889.5	16729.2	7880.2
2	16246.8	3260.4	28300.1	10938.1	5106.0
3	62340.8	13970.7	104751.1	40976.1	21834.3
4	575.7	111.0	1111.1	426.7	234.3
5	5048.8	1113.9	8121.9	3225.7	1544.0
6	14063.5	3312.4	21611.3	8619.6	4535.6
7	161.1	32.2	319.9	121.5	67.2
8	4.5	1.3	6.7	2.6	1.4
9	27.1	4.7	22.0	21.7	5.1
10	32.7	6.0	28.9	16.7	6.8
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.1	0.0	0.0

R68A	46	47	48	49	50
1	26649.1	20099.8	186640.6	61262.3	16772.6
2	16794.0	8743.7	1350190.0	20059.4	8783.0
3	79995.9	31791.5	352509.6	88531.5	22446.4
4	883.5	294.3	3334.5	832.0	190.9
5	5229.5	2386.8	242333.9	6226.5	2352.3
6	16527.3	7362.3	98767.4	21816.5	5430.6
7	244.8	81.8	931.0	191.1	52.6
8	5.3	2.5	23.5	4.6	1.4
9	34.4	14.2	151.3	23.6	9.8
10	38.4	23.7	205.6	28.3	11.3
11	0.0	0.0	0.1	0.0	0.0
12	0.0	0.0	0.2	0.0	0.0

R68A	51	52	53	54	55
1	1310.2	4438.4	1019.1	4901.3	561.4
2	1249.4	2835.7	414.0	2353.4	370.9
3	2625.8	11948.7	1349.9	7028.1	906.3
4	24.8	127.3	10.6	58.0	7.6
5	346.6	891.5	136.5	787.7	118.1
6	660.7	2620.2	344.8	1867.8	248.6
7	6.7	35.3	2.9	15.1	1.9
8	0.3	1.0	0.2	1.0	0.2
9	2.3	3.4	1.2	9.5	1.7
10	2.5	5.1	1.2	9.7	1.7
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

R68A	56	57	58	59	60
1	23728.7	39811.8	49057.5	69608.1	86726.1
2	18543.8	2912.8	7017.6	8167.5	14072.4
3	50689.9	51719.9	65559.1	76149.3	122965.2
4	500.9	449.2	568.3	581.1	1095.3
5	5429.9	1746.0	2808.7	3287.7	5296.2
6	12231.0	10806.6	13724.2	16452.3	25420.9
7	136.5	126.8	163.2	171.2	317.7
8	5.2	0.8	1.5	1.8	3.5
9	37.5	0.7	1.3	1.3	2.5
10	42.2	6.6	6.1	4.0	8.2
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.1	0.0	0.0	0.1

R68A	61	62	63	64	65
1	30111.2	13148.3	585.6	17090.3	16982.5
2	6426.4	2832.8	56.0	5708.6	6450.5
3	45006.2	21490.6	835.5	28495.4	27682.9
4	417.7	200.7	7.6	263.9	257.8
5	2147.0	996.4	26.6	1780.1	1918.5
6	9629.8	4402.4	165.5	6057.8	5753.7
7	114.0	58.1	2.2	76.4	75.0
8	1.6	1.0	0.0	1.7	1.2
9	1.0	0.5	0.0	1.5	1.5
10	3.0	1.6	0.0	3.5	7.4
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

R68A	66	67	68	69	70
1	85330.3	26955.6	9386.1	7319.5	4435.2
2	27401.8	9527.4	3268.7	6881.4	445.2
3	125247.8	38858.6	9039.8	18700.4	3267.0
4	1096.6	333.8	10.1	164.8	12.5
5	8220.9	2864.3	980.9	2482.8	223.1
6	27153.6	8439.0	1115.5	4211.2	835.0
7	319.7	98.4	3.9	51.5	4.2
8	7.5	2.0	0.8	2.6	0.3
9	4.6	2.9	169.7	7.9	0.3
10	12.1	7.7	1.9	6.7	0.8
11	0.0	0.0	0.0	0.0	0.0
12	0.1	0.0	0.0	0.0	0.0

R68A	71	72	73	74	75
1	14671.5	19776.6	5684.2	84053.0	2582.0
2	7917.3	31565.2	12154.0	3829.1	357.1
3	22773.3	34395.8	12710.8	67040.9	2465.3
4	202.6	310.2	124.1	384.3	16.7
5	2309.7	7404.3	2819.4	2407.0	134.4
6	5357.5	8548.8	3080.5	15691.9	563.1
7	53.6	39.9	36.0	110.6	4.9
8	2.1	2.4	0.9	1.0	0.1
9	2.4	4.8	2.4	1.0	0.1
10	8.2	7.8	2.8	2.4	0.2
11	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0

R68A	76	77	78	79	80
1	36035.8	80486.3	39487.4	4930.1	49847.0
2	5244.0	20155.2	7079.0	2457.5	13647.2
3	48206.2	108060.9	50836.4	12368.6	60318.4
4	416.2	875.6	423.6	128.7	462.2
5	2024.7	6430.4	2614.4	811.5	4649.5
6	10070.9	24027.7	10795.4	2448.0	13894.8
7	121.1	259.8	124.6	37.7	135.4
8	1.7	10.8	1.7	0.6	4.2
9	1.2	3.1	2.2	0.9	5.8
10	2.8	6.5	6.4	2.5	13.6
11	0.0	0.0	0.0	0.0	0.1
12	0.0	0.0	0.1	0.0	0.1

R68A	81	82	83	84	85
1	296455.4	27348.3	15154.8	2419.1	9403.8
2	990366.7	45413.9	53065.9	6092.4	18792.6
3	804865.4	45762.8	130363.9	11324.2	211236.9
4	7458.5	226.8	334.9	48.3	119.0
5	216516.1	22906.6	15511.2	1569.1	13753.4
6	203380.9	73786.3	39334.2	3125.9	75853.7
7	2131.2	89.6	132.1	13.0	316.9
8	85.5	13.4	29.3	0.6	336.2
9	1029.5	19.4	17.4	5.1	12.2
10	281.6	270.0	30.3	4.7	9.4
11	0.2	0.0	0.1	0.0	0.0
12	0.3	0.0	0.1	0.0	0.0

R68A	86	87	88	89	90
1	33902.3	12244.6	6859.3	203677.8	129852.3
2	19518.6	33274.9	4290.8	85332.0	60509.4
3	56739.9	30992.3	22268.9	397468.9	305216.1
4	479.9	193.7	88.6	2826.7	2742.8
5	7325.4	16214.2	1950.7	36890.8	32621.2
6	12523.9	10485.4	6750.2	97984.9	69139.5
7	144.0	67.0	42.0	923.5	832.5
8	3.5	2.5	20.1	119.9	45.0
9	29.2	10.7	7.0	67.9	31.2
10	25.4	12.4	5.6	119.5	73.0
11	0.0	0.0	0.0	0.2	0.1
12	0.1	0.0	0.0	0.3	0.2

TABLE A.9 : B VECTORS

BI68	1	2	3	4	5
	3482212.0	4229598.0	6740971.0	53030.4	1081419.0
BI68	6	7	8	9	10
	1532972.0	15068.5	925.3	2435.3	3779.8
BI68	11	12			
	3.9	5.5			
BC68	1	2	3	4	5
	221024.9	1547869.0	749470.7	4112.7	244152.3
BC68	6	7	8	9	10
	105008.1	358.6	0.0	0.0	0.0
BC68	11	12			
	0.0	0.0			
BT68	1	2	3	4	5
	3703236.0	5777467.0	7490441.0	57143.1	1325571.0
BT68	6	7	8	9	10
	1637980.0	15427.1	925.3	2435.3	3779.8
BT68	11	12			
	3.9	5.5			

TABLE A.10

Rankings of Commodities According to Pollutant (Unit) Impact (SI)

[Higher numbers in the ranking indicate lower pollution]

Order \ *Ecologic Commodity	1	2	3	4	5	6	7
1	55	75	63	63	63	63	55
2	63	55	55	55	55	55	63
3	53	53	53	68	75	53	26
4	51	70	26	53	53	26	53
5	28	26	75	70	70	75	68
6	84	28	51	26	26	51	70
7	26	24	28	75	51	28	75
8	75	51	70	28	28	70	28
9	10	10	54	51	10	68	51
10	04	54	66	24	54	54	84
11	36	79	30	84	79	05	54
12	70	62	05	54	05	30	24
13	52	52	84	03	52	79	15
14	54	57	52	30	24	36	30
15	79	42	36	88	68	52	29
16	02	68	79	29	62	10	05
17	30	05	93	05	42	04	42
18	73	33	29	42	33	20	52
19	05	19	42	06	36	73	06
20	33	74	33	85	19	84	73
21	20	23	20	73	30	33	36
22	88	20	04	52	20	29	79
23	69	88	24	36	45	19	19
24	45	29	19	79	84	42	33
25	19	22	06	19	29	24	32
26	68	30	10	20	57	06	20
27	85	36	32	33	64	69	88
28	32	45	69	32	23	32	34
29	29	76	34	04	65	62	04
30	42	14	62	69	14	45	69
31	16	64	02	34	88	34	50
32	87	21	45	50	32	71	71
33	06	84	88	87	76	50	31
34	62	61	50	10	34	31	10
35	34	65	71	62	04	12	62
36	12	02	31	71	61	65	02
37	31	32	12	31	71	64	23
38	71	69	65	02	50	88	67
39	83	34	14	82	21	14	45

(cont.)

TABLE A.10
(cont.)

40	44	58	87	12	74	21	12
41	50	76	47	23	69	47	65
42	65	71	14	45	76	02	64
43	64	59	21	65	06	25	03
44	40	47	72	64	22	40	47
45	14	50	25	25	31	67	25
46	72	04	40	21	58	72	21
47	47	31	23	47	73	44	14
48	22	67	67	14	67	23	62
49	39	08	44	72	47	39	72
50	24	63	39	83	44	61	67
51	82	18	61	67	39	76	40
52	21	39	82	40	59	22	27
53	56	06	76	74	08	87	74
54	25	44	18	76	18	18	61
55	46	16	56	61	40	76	39
56	67	73	78	39	12	57	76
57	35	25	57	78	25	56	44
58	38	80	22	44	80	86	78
59	61	40	35	18	09	35	57
60	18	60	86	57	41	58	18
61	86	17	27	80	46	80	83
62	76	09	80	09	60	41	80
63	41	41	38	86	56	38	56
64	43	46	41	27	49	74	09
65	78	12	58	56	35	09	86
66	57	56	74	35	77	17	35
67	27	85	09	58	86	59	38
68	23	86	37	17	72	46	41
69	17	49	17	41	43	27	58
70	37	77	59	59	66	15	59
71	58	35	46	22	38	37	22
72	80	27	49	38	27	08	37
73	07	66	08	08	17	43	17
74	49	43	43	37	37	49	08
75	15	38	77	49	13	16	49
76	09	72	07	16	02	77	46
77	59	13	60	77	85	60	77
78	77	87	66	46	83	07	01
79	74	37	83	01	87	66	85
80	66	82	01	07	07	83	60
81	60	83	16	60	82	13	66
82	08	90	15	66	15	01	43
83	13	89	13	43	90	82	07
84	90	07	85	13	89	03	16
85	01	01	90	90	01	90	13
86	03	11	48	89	11	85	90
87	48	15	11	48	16	48	89
88	89	03	89	15	03	11	48
89	11	48	03	11	48	89	11
90	81	81	81	81	81	81	81

*For detailed list see Table 4.1 (pp.174-6)

TABLE A.11

Rankings of Commodities According to
Ecologic Magnitude or Total Impact (RI)

[Higher numbers in the ranking indicate lower pollution]

Order \ *Ecologic Commodity	1	2	3	4	5	6	7
1	87	88	87	87	88	87	87
2	88	90	88	88	90	90	88
3	83	89	90	16	14	88	03
4	90	14	03	03	52	03	90
5	03	52	29	83	89	29	84
6	84	87	89	90	29	52	29
7	52	29	52	68	03	89	89
8	16	16	86	89	87	86	83
9	89	86	28	29	49	28	68
10	73	28	73	15	28	14	28
11	85	03	51	84	43	73	86
12	82	49	14	85	86	40	52
13	51	42	40	82	40	51	82
14	86	57	06	28	67	06	26
15	40	19	31	86	42	31	06
16	43	22	38	06	46	43	51
17	29	43	72	52	66	72	14
18	02	60	82	14	51	38	73
19	28	46	84	73	19	35	15
20	14	58	32	51	77	32	31
21	81	40	30	01	45	19	40
22	69	67	37	40	31	37	01
23	46	59	35	31	60	34	32
24	36	66	34	72	41	69	72
25	38	18	43	30	34	84	30
26	35	62	01	32	62	67	49
27	72	77	42	42	18	30	38
28	48	45	49	38	32	42	34
29	31	41	19	53	33	36	37
30	30	76	69	37	35	49	83
31	32	47	53	34	47	46	54
32	56	51	54	26	58	25	42
33	37	33	81	54	53	47	35
34	33	61	67	19	61	53	55
35	44	53	83	69	64	41	19
36	06	31	47	35	39	01	27
37	19	78	36	25	59	45	67
38	34	34	25	49	79	81	25
39	39	17	55	67	44	66	69

(cont.)

TABLE A.11
(cont.)

40	07	75	26	55	36	18	81
41	79	79	56	43	38	54	43
42	10	21	41	81	37	33	47
43	49	32	46	47	48	56	41
44	48	64	66	41	80	82	56
45	47	80	33	66	65	39	66
46	55	39	07	18	84	44	80
47	41	54	45	80	54	07	18
48	20	35	18	56	09	48	36
49	54	48	39	09	78	55	09
50	67	09	48	36	76	26	33
51	42	69	44	77	06	20	77
52	53	44	11	27	75	15	07
53	25	26	27	07	56	11	16
54	18	74	80	33	13	79	46
55	66	65	77	17	30	77	39
56	12	23	02	11	83	83	48
57	11	13	12	70	69	80	11
58	01	25	20	46	21	65	45
59	27	37	17	39	25	64	50
60	04	24	65	48	73	17	17
61	17	38	79	45	22	12	44
62	64	56	09	50	57	68	12
63	65	36	64	44	10	09	70
64	22	84	50	12	20	16	64
65	77	30	16	02	55	21	75
66	80	20	21	75	72	50	02
67	50	83	62	65	50	27	65
68	62	02	61	64	23	62	20
69	21	70	60	20	26	60	21
70	13	55	75	21	17	61	79
71	61	10	78	24	08	78	61
72	60	06	58	79	27	13	24
73	15	08	13	59	07	58	60
74	09	63	59	78	11	75	62
75	26	73	15	60	70	22	78
76	71	27	71	62	74	59	59
77	78	85	76	61	12	02	58
78	58	82	22	58	71	76	71
79	76	50	68	23	63	71	23
80	59	72	70	13	81	10	13
81	75	15	04	76	24	04	76
82	05	11	23	71	68	23	22
83	68	71	10	22	82	70	08
84	57	07	57	08	01	57	57
85	23	68	08	57	05	05	85
86	70	12	05	74	85	08	74
87	08	81	24	04	04	24	04
88	63	01	74	05	02	74	05
89	24	05	85	10	16	63	10
90	74	04	63	63	15	85	63

*For detailed list see Table 4.1 (pp.174-6)

CHAPTER 4:

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